A Fine Conceptual Analysis Needs No "Ism"

Gerald A. Goldin
Rutgers University, USA
geraldgoldin/at/dimacs.rutgers.edu

> Upshot • The key philosophical premise of von Glasersfeld's radical constructivism is not necessary to the insightful conceptual analysis presented by Cifarelli and Sevim, which could benefit from abandoning it.

« 1 » As Victor Cifarelli and Volkan Sevim note (§1), what makes Ernst von Glasersfeld's constructivism radical is his *a priori* rejection of any interpretation of knowledge as objective. Indeed,

> "Constructivism drops the requirement that knowledge be 'true' in the sense that it should match an objective reality. All it requires of knowledge is that it be viable, in that it fits into the world of the knower's experience, the only 'reality' accessible to human reason." (Glasersfeld 1996: 310)

« 2 » This premise has drawn intense criticism from many educational researchers, myself included (Goldin 2003a), as having negative consequences for both mathematics education research and practice. First, inferences drawn from it limit severely the constructs that can be used to understand and interpret empirical observations of the kind reported by Cifarelli and Sevim. Second, by devaluing truth and objectivity and replacing validity by viability, it greatly diminishes the influence that important empirical findings can have on education policy.

« 3 » However, radical constructivists' qualitative research and analyses have provided, and continue to provide, some extremely valuable exploratory and descriptive findings. These should tend to support policies and practices that are far more progressive and sophisticated than those being implemented now across the United States as test-score-oriented goals for school mathematics are increasingly mandated. Research findings, as their validity and generalizability are established, point toward placing greater emphasis on relational understanding relative to instrumental understanding (Skemp 1976), and toward more complex and more qualitative assessment methods.

« 4 » Thus, as I have maintained for some time (Goldin 2003b), it is long overdue for us to question whether the methods, findings, and analyses in this research actually *require* acceptance of RC's ultrarelativist epistemology. I would argue that much of the research is equally compatible with scientific perspectives generally taken in mathematics, physics, cognitive science, and empirical psychology. Although I am fully aware that many constructivist researchers regard RC as essential to their programs of research, in my view it actually limits the research severely.

« 5 » The present report by Cifarelli and Sevim serves as an example for consideration. They present an excellent, fine-grained conceptual analysis of problem-solving activity by one student, Marie, based on von Glasersfeld's notion of re-presentation, attending especially to her reflections (§22) and to her conceptual structures and anticipation (§23). They do more than provide important empirical evidence of the complexity of Marie's learning and problem solving. They also suggest some valuable theoretical distinctions that may help one to understand their observations, with wider implications if the inferences they suggest are validly generalizable to other learners.

« 6 » With this in mind, I read carefully the conceptual analysis of Marie's problem solving activity, asking throughout whether one needs RC epistemology at all to make the observations or to carry out the analysis. I think the conclusion is straightforward. The choice that Cifarelli and Sevim make to study "idiosyncratic internal constructions, which … are formed by re-presentation of prior experiences" (§50) was clearly *motivated* by the claim that they "do not stand for any objects." But it does not in any way *depend* on that claim. The study of "re-presentation," while *inspired* by von Glasersfeld, need not entail adoption of the view that this is the *only* possible kind of representing relationship.

« 7 » One does not require RC as one's motivation to consider Marie's internal constructions, reconstructions, reflection processes, anticipations, and representations as worthwhile theoretical constructs, or to draw inferences about them from her observed statements, behavior, and productions. One can do *quite the same* analysis of Marie's problem solving without von Glasersfeld's radical tenet. And without RC, there is no need for or value in rejecting conventional notions of truth, correctness, correspondences between internal and external representations, and so forth. The way is then clear to introducing additional theoretical constructs regarding representation into the analysis (Goldin 1998; Goldin 2003b).

« 8 » For example, one may seek to characterize structures in external, real-world objects, configurations, representations, and systems of representation, and allow the examination of dynamically evolving, two-way representing relationships between the internal and the external. Let us elaborate briefly on this.

« 9 » Without the RC tenet, Marie's own inscriptions – equations, diagrams, numerals, and written words reproduced pictorially in §§25, 26, 27, 29, 32, 36, and 38 – may usefully be considered as real-world objects about which Marie has partial knowledge. In these inscriptions, one may characterize semantic relationships beyond the meanings that Marie herself might attribute to them at the time she created them: relationships that are "there to be discovered," and that Marie may or may not in fact later discover.

« 10 » In addition, one may consider real-world lakes, altitudes, cities, distances, storage drums, and so forth as objects of the kind that the problems are "about." These have attributes and structural properties that exist *apart* from Marie's experiential world. Some of Marie's re-presentations have to do with her partial representation of those real world properties – again, relationships are "there to be discovered."

« 11 » External representational systems also include standard mathematical and pedagogical systems, such as our systems of base ten numeration and algebraic notation. These are based, of course, on socially-agreed conventions; but having been created, they incorporate mathematical relationships "outside the mind" – in particular, outside Marie's mind. Nevertheless, some of her mental processes can be usefully examined with respect to such relationships. When major discrepancies occur, the notion of "misconception" rejected in §49 becomes a valuable descriptive construct.

« 12 » Most important, in addition to augmenting the theoretical possibilities for understanding learning and problem solving, abandonment of the "ism" opens the door to achieving conventional, time-tested scientific goals. Mathematical cognition and learning themselves incorporate *real-world* phenomena, about which we seek *objective*, *scientifically valid* knowledge. This is gained by replicating and generalizing exploratory findings, testing and confirming or disconfirming hypotheses, validating inferences drawn, and so forth.

« 13 » In short, the fine conceptual analysis in this article needs no "ism" to justify itself or to motivate its constructs. Rather, it may be regarded as a valuable contribution to the exploratory and descriptive stage in the enterprise of conducting objective scientific research.

Gerald Goldin, Distinguished Professor of Mathematics Education, received his Ph.D. in physics from Princeton and studied mathematics education at Penn, He co-chaired the PME Working Group on Representations, organized New Jersey's Statewide Systemic Initiative, and directed two research centers at Rutgers. He publishes on education, mathematics, and physics, and is a Humboldt Prize awardee. His current investigations include mathematical affect and engagement.

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Convergences between Radical Constructivism and Critical Learning Theory

Karen François
Vrije Universiteit Brussel, Belgium
karen.francois/at/vub.ac.be

> Upshot • The value of Cifarelli & Sevim's target article lies in the analysis of how reflective abstraction contributes to the description of mathematical learning through problem solving. The additional value of the article lies in its emphasis of some aspects of the learning process that goes beyond radical constructivist learning theory. I will look for common ground between the humanist philosophy of mathematics and radical constructivism. By doing so, I want to stress two converging elements: (i) the move away from traditionalist ontological positions and (ii) the central role of the students' activity in the learning process.

« 1 » The authors' examination of the role of re-presentation in mathematical problem solving as an application of Ernst von Glasersfeld's conceptual analysis serves to connect the philosophical discussion on knowing and learning and the philosophical debate on the ontological status of things (§1). They do not engage in a philosophical examination of Glasersfeld's view of constructivism as radical and instead refer to his 1984 paper, in which von Glasersfeld takes a position in the ontological debate (§1). To von Glasersfeld (1984), knowledge does not reflect an ontological reality. Instead, it results from an ordering and organization of experiential reality. The authors (§5) focus on the second part of von Glasersfeld's non-ontological definition of knowledge and do not consider the relation between the ontological assumption and its possible impact on the learning process. They bridge the philosophical discourse and the conceptual analysis of a goal-directed activity in the context of problem solving by concentrating on the constitution of mathematical knowledge by experience, leaving aside the ontological issue. The authors go beyond ontology, and focus on the cognitive actions of the learner in resolving mathematical problems.

« 2 » It was Reuben Hersh (1997) who argued for the connection between the philosophy of mathematics and the teaching of it. Hersh stated that the ontological position of a mathematics teacher or the ontological position of mathematics as reflected in the curriculum influences the learning process. Humanist philosophy of mathematics takes the teachability of mathematics as a central concern. Hersh (1997: 182) calls his "own slant on humanist philosophy of mathematics" "social-cultural-historic" or just "social-historic." He tries to link up mathematics with people, society, culture and history. Instead of a Platonist ontology, a humanist conception of mathematics brings mathematics back to Earth as a human activity that is embedded in a historical and cultural environment. It narrows the gap between students and the subject matter so that students can realize that they are taking part in the practice of mathematics. Students can learn and understand mathematics and they can elaborate on it, going from practical basic skills to theoretical abstract reasoning.

« 3 » The humanist philosophy of mathematics and of mathematics education is fully adopted by critical learning theories. These theories emphasise the learning of mathematics as a right for all students irrespective of their social and cultural background. The learning and knowing of mathematics is seen as a stepping stone for a future career, for the development of society, and as a basis for an informed and critical citizenship. The mathematician and educator Ubiratan D'Ambrosio – who can be seen as the founding father of ethnomathematics – pleads for educational reform and for more attention to students and teachers as human beings. For D'Ambrosio (1990), we have to realize that mathematics and other scientific disciplines are epistemological systems that are embedded in their socio-cultural and historical perspective. These bodies of knowledge are not finished and thus do not consist of static entities of results and rules. From a humanist philosophical point of view, teachers should accept, understand and access their students' social and cultural background knowledge as a starting point for the learning and doing of mathematics. Critical voices from the research field of mathematics education (such as Hersh & John-Steiner 2011) propose to look at mathematics as the study of mathematical activities, and hence as a complex of social and cultural manifestations, in a move away from traditionalist positions (e.g., against the still dominant Platonic view).

« 4 » For these critical voices, the theoretical starting point is the recognition of a factual variety of mathematical practices, including "western" academic mathematics. "Western" academic mathematics has a particular setup of concepts and practices of and about the world and thus is, at its basis, a sociocultural complex like any other cultural tradition. This point of departure is consciously and explicitly taken into account when devising curriculum material and educational strategies by researchers from critical research fields in mathematics education. It is of great importance in the learning process that students will understand and will take insightful steps toward