"Mathematical" Schemes as Instruments of Interaction

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1 Elsewhere, Ernst von Glasersfeld draws from Piaget's (1954) studies of the child's construction of reality as a corroborating indication of the viability of his attentional model as an explanation of children's construction of permanent object concepts. In his current paper, von Glasersfeld introduces the possibility of extending the use of his attentional model in explaining children's construction of others and social interaction. Von Glasersfeld's hypothesis is that his attentional model can be used in explaining how children construct a category of self-moving things (animals) and others not unlike themselves. In this construction, "In order to develop relatively reliable schemes, the child must impute certain capabilities to the objects of interaction. But now these ascriptions comprise not only perceptual but also cognitive capabilities, and these others will be seen as intending, making plans, and being very predictable in some respects and not at all in others" (§19).

2 Von Glasersfeld's hypothesis that children impute cognitive capabilities to those with whom they are in interaction opens up a crucial line of research in the mathematics education of children regarding how children use their "mathematical" schemes[1] as instruments of interaction with their peers and with their teachers. One of the main problems that plague studies of children's mathematical interaction is that researchers who are primarily interested in social processes seldom provide accounts of the schemes of action and operation that the interacting children use and how the children use them as they interact with each other and with their teachers concerning mathematical situations. As a consequence, there is no account provided of the meanings that individual children construct in what an observer construes as mathematical interaction, nor is there an account given of individual children's mathematical learning either during or across interactive episodes. My goal in this commentary is to say enough to suggest that the meanings children impute to the language and actions of other children are based on their current conceptual schemes and that, if the schemes are at different levels of the constructive process, it is no easy feat for children to use their schemes in interactive mathematical communication.

Establishing meaning in interaction

3 Von Glasersfeld, citing Shannon, reminds us that, "the sounds of speech or the visual patterns of print or writing in linguistic interactions – do not actually carry or contain what we think of as *meaning*. Instead, they should be considered as instructions to select particular meanings from a list that, together with the list of agreed signals, constitutes the "code" of the particular communication system" (§25). Mathematical interaction, of course, is often not just linguistic interaction, since it might include children's interaction with items such as geometrical sketches, computer icons, or modified forms of the abacus. Regardless of what items (the observer's items) or what sensory channels are involved in an interaction, according to von Glasersfeld (§26), the individual must construct the meaning of what is sensed. The construction of meaning involves cognitive assimilation, which Piaget (1964, p. 18) regarded as, "the integration of any sort of reality[2] into a structure." I interpret "structure" in the citation from Piaget in terms of the schemes that children use in mathematical interaction and the *meaning* of what is sensed as those aspects of the schemes that are within the awareness of the assimilating child.

4 In studies of children's mathematical interaction, it is essential to remember that the meanings children might attribute to the items they establish are inferences of the observer. In my experience of analyzing children's mathematical schemes, although there was often good reason to infer that they were aware of the *results* of operating, it seemed as if the operations that I inferred the children used were outside of the grasp of their consciousness. I have always found that occasioning the metamorphoses that make such awareness possible in children is difficult. But when such monumental events of learning do occur, children open new mathematical frontiers and they find that what was previously difficult is now easy. But I am not only concerned with attempting to help children become aware of their operations. In fact, my primary concern lies in researchers becoming aware of the operations that *they* use to construct children's mathematical schemes and how children use those schemes in interaction.

Operations and interaction

5 Elements of the researcher's explanatory models of children's mathematical schemes serve as analytical constructs when accounting for meanings that children establish in mathematical interaction. As an illustration, consider two 5th grade children, Jason and Laura, engaging in interaction using a computer tool called TIMA: Sticks[3] (Steffe 2003). At the start of the school year, I inferred that Jason had constructed what I call recursive partitioning operations,[4] whereas Laura seemed to have not yet constructed them. In Figure 1, Laura had partitioned a stick into four parts, pulled out one part, partitioned that part into four parts, colored three of them, and pulled out the three colored parts while playing in TIMA: Sticks.[5] When asked what fractional part the three parts was of the original stick, Laura guessed "three-tenths." After resting his chin on his hand and thinking, Jason answered, "three-sixteenths." In explanation, he pointed to the three parts and said, "See, if we would have had it in that, four, four, four, and four – sixteen. But you colored three, so it is three-sixteenth!" It was Jason's goal to find what fractional part of the 4-part stick the 3-part stick comprised, and to do so he introduced partitioning each part of the 4-part stick into four parts. That is, he engaged in recursive partitioning. Laura, on the other hand, did not engage in recursive partitioning.

6 The qualitative difference in the operations of the two children can be, in part, understood by inferring the unit structure that Jason established prior to his explanation. There is good reason to infer that he then established sixteen as a composite unit containing four other composite units, each of which contained four units. That is, I infer that he established a unit of units of units as a result of recursive partitioning. There was nothing in Laura's guess of "three tenths" that I could use to infer her operations, so I looked

to another task where the children were asked to partition a stick into 12 parts using Parts, but they were not to dial Parts to 12 to infer Laura's partitioning operations. Laura dialed Parts to 11, pulled out one part, and joined it to the end of the stick, establishing a 12-part stick. Jason, on the other hand, drew a new stick and insightfully partitioned that stick into three parts and then each part into four parts using recursive partitioning operations, which corroborates how he operated in the above task.

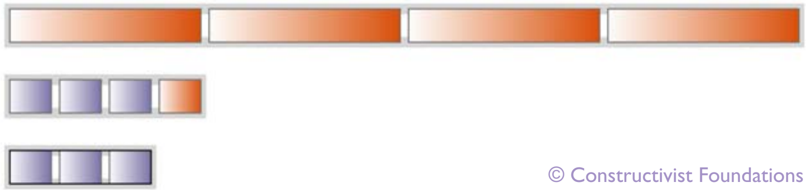

Figure 1: The result of Laura's partitioning actions.

7 I infer that Laura used her number concept, eleven, as a partitioning template. My inference is that Laura produced two levels of units – a unit that contained the 12-part stick and the units that constituted the 12 parts. She used the operations that produce a unit of units in partitioning, but not the operations that produce three levels of units. Again, there was no indication that she could engage in recursive partitioning, which involves using the operations that produce three levels of units.

8 Laura did offer the explanation that Jason had "three pieces and added four in each thing." In the explanation, she *reenacted* Jason's operations by coordinating partitioning the stick into three parts and then each part into four parts. I call the operations she used to reenact Jason's operations "units-coordinating operations." I infer that she was definitely aware of a *sequence* of three units, each containing four parts in her visual field, and that this awareness constituted her meaning of the results of Jason's actions. On the other hand, I infer that Jason anticipated partitioning the stick into 12 parts prior to action and that he was aware of an image of three levels of units, however amorphous or minimal the image might have been. I make the same kind of inference about Laura in the case of two levels of units, but not three levels of units. In the latter case, she could only operate on material in her visual field. But in that case, there was the issue of whether she could only produce a sequence of composite units or whether she could take the sequence as a unit.

9 The difference in the partitioning operations of the two children had major consequences in their subsequent mathematical interactions. For example, the teacher/researcher explored interactions concerning commensurate fractions[6] following the children's attempts to partition a stick into twelve parts without dialing Parts to "12" (Steffe 2003). During the exploration, Laura made a 15-part stick by first partitioning a stick into three parts then each part into five parts using her units-coordinating operations.[7] The teacher/researcher then pulled three parts out of the 15-part stick and Laura said that the 3-part stick was three-fifteenths of the 15-part stick. So, the teacher/researcher decided to tell the children that the 3-part stick was also one-fifth of the 15-part stick to find out if the children could provide an explanation. Laura said that she did not agree that it could be one-fifth. Jason, however, explained that it took five of the 3-part sticks to make the 15-part stick. In explanation, he pulled a 3-part stick out of the 15-part stick, made copies of it, and aligned five of them directly beneath the 15-part stick. Laura then said, "I get it!" Her saying that she got it, however, did not indicate that she had established the operations that produce commensurate fractions. The reason is that immediately after saying, "I get it," she looked disconcerted when she measured another 3-part stick and "1/5" appeared in a number box. She could not explain why "1/5" appeared and, for her, the situation was unrelated to the immediately prior situation. So, her saying "I get it" indicated that she established the relation using the units she established in her visual field and as soon as the perceptual situation was not available to her, the relation was also unavailable. The relation was ephemeral and occurred only in the immediate here-and-now.

10 Laura had yet to construct the operations that produce three levels of units in the absence of perceptual material. As a consequence, the two children could not communicate *about commensurate fractions* if that means, as Maturana & Varela (1980) suggested, that one organism can affect the behavior of another organism by orienting the behavior of the other organism to some part of its domain of interactions *comparable* to those of the orienting organism. "This can take place only if the domains of interactions of the two organisms are widely coincident" (Maturana & Varela 1980, pp. 27–28). Jason's language and actions did orient Laura to interact with the results of his actions, but I do not consider that these interactions between the two children constituted communication in the sense that Maturana & Varela explained[8]. The contribution of non-communicative interactions to the construction and/or modification of mathematical schemes is but one among the most critical problems in the line of research in mathematics education that is opened up by von Glasersfeld's hypothesis that addresses the individual in mathematical interaction.

Notes

1. I use quotation marks to indicate that the schemes to which I refer are abstracted from observations of children's interactions that I judge as mathematical. In the remainder of the text, I drop the quotation marks for convenience of reading.
2. I interpret "any sort of reality" as the observer's reality.
3. TIMA is an acronym for Tools for Interactive Mathematical Activity.
4. Partitioning operations are recursive if a child can partition a given partition in service of a non-partitioning goal.
5. In TIMA: Sticks, a child can draw a stick, partition the stick into as many parts as desired up to 99 by selecting a computer action PARTS, dialing a counter to any number through 99, and then clicking on the stick using the mouse. Using PULL-OUT, the child can then click on one or

more of the parts produced and pull them out of the stick while leaving the stick intact. The child can also fill any part with any one of 12 colors.

6. I use "commensurate" to indicate a relation between, say, one-third and five-fifteenths that children produce by partitioning a segment into three parts, partitioning each of the three parts into five parts, pulling one of the three partitioned parts out from the three original parts, and establishing it as one-third as well as five-fifteenths. When children can operate similarly in the absence of perceptual material, I refer to the relation using "equal fractions."
7. When it was Laura's goal to make a partition of a partition, she could do so using her units-coordinating operations. But that does not constitute recursive partitioning.
8. When the children each constructed two levels of units, they did engage in interactive mathematical communication.

"Things That Go Bump in the Night"

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1 Here I underline two features of radical constructivism (RC) that render it more difficult to "cross-over" into the "social." Firstly, the fact that we, as observers, do not have any privileged access to "*external objective reality*," and secondly, that we do not have any privileged access to what we call our "*internal subjective reality*" either.

2 My comments focus on §4 and those following it, which deal with the "Seeing is believing" cliché. My concern is to elaborate the RC position that we cannot have a privileged access to external reality. Apart from the case of "vision," I have noted a temptation for writers to attribute a special privilege to the sense of touch, as if it gives us "direct access to reality." I then go on to apply this analysis to the assumption that we have a privileged access to our *own* inner experiences. My aim is to clarify that for both "external reality" and for "internal reality" we remain without any privileged access.

3 In §4 Ernst von Glasersfeld outlines the RC position regarding the fact that "vision" has no direct access to objective reality (as is implied in the common phrase "Seeing is believing"). There is sometimes a tendency to quietly privilege "touch" with having a direct access to reality because, while most people can recognise that their "eyesight" may play tricks on them from time to time, it seems that when we "bump into objects" in our environment there can be no mistake about it – and that therefore we have been in "direct contact with reality," and "know" that we have hit up against a "constraint" or an "obstacle," which tells us that our current pathway is not viable. (It is not entirely coincidental that realists choose exactly this area to demonstrate the "silliness" of idealists who are asked to deny that they feel pain when they walk into a glass door!)

4 However, we cannot *know* that we have "bumped into some object" or "constraint" just from the "bumping." We cannot *know* that it *is* a barrier or "impediment." It is not the "world" that tells me (via the bumped-into object) that there is a constraint that I have not succeeded in circumnavigating. It is always necessary for one to construct a picture of the world out of the sense signals which arrive together in one's experiencing. It is "I" who must construct my experience as that of "encountering a constraint." That is, I construe my experience of the situation as having felt a limit to the operational usefulness of my understanding.

5 The impression that the sense of touch gives us a privileged access to objective reality is an illusion: from the RC point of view all of the senses have the same epistemological status as the sense of vision – i.e., none of them can give us a privileged access to reality. Maturana's point that at the moment of experiencing an event we cannot tell an "illusion" from a "perception" (Maturana 1988) applies to all of our senses – not just to "tricks of the eye." We may experience not only hallucinations of "seeing," but also of hearing, smelling, tasting and touching.

6 The phrase used by Ernst von Glasersfeld in §4, "Seeing is believing," can be equally used for the other senses thus: "Hearing is believing," "Smelling is believing," "Touching is believing," "Tasting is believing." In effect, the problems presented in psychotherapy can be read as the inversion of the "Seeing is believing" phrase, which illustrates in part why people get into difficulty in their lives and why often they find it so arduous to solve their problems. In psychotherapy, therefore, we frequently see phenomena arising from the inverse conviction to that of the realist ("I'll believe it when I see it") position. In contrast to this view we have many problems presented in psychotherapy that are well represented by the phrase "*I'll see it when I believe it*" ("Believing is seeing") as also with the other senses, such as "*I'll hear it when I believe it*" ("Believing is hearing") etc. One of the crucial tasks of the therapist is to understand what the person is *able to listen to* and *hear*.

7 While it is clear that the RC position says that we cannot have any privileged access to outer objective reality, it is less clear – but I would claim it is nonetheless the case – that neither can we have any privileged access to what we regard as our own *inner reality* – our interior world of experiencing.

8 Allowing that the five senses do not give us a privileged access to the outer world, it also needs to be said that neither do they give us a privileged access to any *inner world* of experiencing that the senses generate in our body.

9 The fact that it is *our* own body that manifests "our experience" (that which cannot be "shared with" or "compared with" that of others) does not mean that we ourselves have a privileged access to "*our own experience*." As Maturana repeats, we live in two different and non-collapsible domains – the domain of experience and the domain of explanations (Maturana 1990). The domain of experience is not translatable or reducible to that of explanations (knowing). There is always a gap between the two domains. This places us as a "knowing observer" across from, and apart from, not only external reality but also from our own ongoing flux of inner experiencing.