analysing man–machine interaction. Theory, Culture & Society 37(5): 27–49

Thom R. (2009) Prédire n'est pas expliquer [Predicting is not explaining]. Flammarion, Paris. Originally published in 1991.

Varela F. J. (1975) A calculus for self-reference. International Journal of General Systems 2: 5–24. ▸ https://cepa.info/1660

Varela F. J., Maturana H. R. & Uribe R. (1974) Autopoiesis: The organization of living systems, its characterization and a model. Biosystems 5(4): 187–196. ▸ https://cepa.info/546

Whitehead A. N. (1978) Process and reality: An essay in cosmology. The Free Press, New York. Originally published in 1929.

Wolfram S. (2002) A new kind of science. Wolfram Media, Champaign IL.

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Open Peer Commentaries

on Paolo Totaro & Domenico Ninno's "The Cognitive Origin of the Continuum and the Discrete"

Self-Production Is Not Self-Reference

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> Abstract • In their article, Totaro and Ninno attempt to solve the problem of the continuum, using Maturana and Varela's theory of autopoiesis and biology of cognition to draw parallels with abstract mathematical number and set theories. All of these are held to involve recursive, self-production, self-referential processes and operations such that they can be regarded as isomorphic to one another. However, the cited mathematical theories they cite involve unobservable platonic-realist entities that bear little or no relation to concrete, limited, embodied observers, biological organisms, or cognitive-psychological processes in minds and brains. Fundamental problems in Maturana's theory of observers and of consciousness are discussed. The proposed approach conflates processes of recursion, self-production, self-reference, and observation, thereby eliminating critical distinctions between embodied, observable processes and platonic mathematical ideas.

Handling Editor • Alexander Riegler

« 1 » Coming from analytic philosophy, in their target article, Paolo Totaro and Domenico Ninno compare Humberto Maturana and Francisco Varela (M&V)'s theories of autopoiesis, cognition, observation, and consciousness with number and set theory in mathematics. They all invoke recursion and self-reference, making them in some sense isomorphic. Some basic problems with these analogies involve use of platonic-realist mathematical ideas and conflation of processes of recursion, self-production, self-reference, and self-observation in both imagined and concrete systems.

« 2 » Arguably, analytic philosophy and pure mathematics are bodies of thought perhaps furthest removed from biology, neuroscience, psychology, epistemology, and first-person experience. This makes them unwieldy starting points for developing coherent ontologies (seeking to establish a fundamental "world structure"), epistemologies (observer mechanics, knowledge), and phenomenologies (experiential contents of consciousness).

Problems with infinities

« 3 » Very early on (§1), Totaro and Ninno invoke Cantorian infinities and Dedekind cuts, which appear to me to be pure platonic-realist entities (Cariani 2012). From the perspective of radical constructivism, these are imaginary entities that cannot be observed directly, for two reasons. The first is that they invoke completed infinities and infinite processes of which there are no concrete, natural examples. The second is that infinities themselves, as apart from their cognitive, ideational mental representations and public symbolic token-referents, are incompatible with finite observers. Observers are themselves finite, temporal entities only capable of registering finite, sets of

distinctions at any one time, and therefore incapable of handling completed infinities and infinite processes. The contents of subjective experience are similarly limited. These limitations doom this project from the start. This makes me wonder: In the target article, how should one consider the ontological status of Dedekind's numbers and Cantor's infinities? Mathematical constructivists such as Leopold Kronecker and Henri Poincaré rejected such entities as ill-defined notions. Are these observable entities? Are they useful in any way? Q1

« 4 » A basic tenet of radical constructivism is that we should avoid naive belief in unobservable entities (Cariani 2010) – those ideas and assertions that can be tested should be consciously distinguished from those that cannot (Cariani 2022). So, on what basis should we evaluate the analogy between unobservable platonic ideas and the structure of observers proposed here by Totaro and Ninno? Q2

Alternatives to platonic systems

« 5 » There exist finitist, operationalist approaches to the foundations of mathematics that are compatible with limited observers and radical constructivist constraints (Cariani 2012). These do not generate infinite regresses, paradoxes, and indeterminacies (§§30–33). Set theory can be made compatible with finite observers if sets are defined exclusively in terms of finite, enumerable collections of observer-distinguishable elements (Goodman 1972), i.e., by disallowing self-inclusion and null sets.

« 6 » Arguably, better approaches to the continuum exist that come from natural philosophy: psychology, physics, and theoretical biology. In perception many qualia, (pitch, color, temperature) span apparently continuous ranges of differences. Gestaltist processes of fusion and separation create discrete entities (objects, events). Cognitive, categorical recognitions (letters, words) have both discrete and continuous aspects. There is a whole neuropsychology of numerosity (Dehaene 1997). Debates over atomistic vs. continuous ontologies, realist vs. empiricist, observer-bound constructivist epistemologies, wave–particle dualities, and measurement complementarities in the foundations of mathematics and physics have been ongoing from ancient times to the present. Ernst Cassirer's (1948, 1955) other ideas on observer-mechanics, measurement, Hertzian modelling relations, the nature of symbolic forms, atomism, and the principle of continuity, might be more apropos than those on functions (§10). Herman Weyl's (1949: 30–66) "Number and Continuum, the Infinite" is especially germane in its discussion of the various positions taken by Plato, Aristotle, Dedekind, Cantor, symbolic-formalist David Hilbert and the finitist-intuitionist E. J. Brouwer regarding the construction of numbers. In addition to M&V, other theoretical biologists, such as Howard Pattee (Pattee & Rączaszek-Leonardi 2012), Robert Rosen (1991), and myself (Cariani 2001, 2022, 2023) have discussed related issues: measurement vs. computation operations in observer mechanics, continuous vs. discrete processes, symbols and dynamics in organisms and brains, cell psychology, semantic closure, and neurophenomenology.

« 7 » Aristotle's thinking (§9) is highly coherent, but open to differing interpretations, mostly due to its near-universal span of concerns. "Aristotelianism" can refer to Aristotle's own ideas, e.g., about logic, or to various medieval distortions that mixed in Platonic and theological precepts. Later interpretations (Graham 1987; Modrak 1987) explore much more sophisticated views, including Aristotle's conception of the continuum (Feyerabend 1987). Daniel Graham argues that the early Aristotle adopted a realist view (properties of objects are external to observers), whereas the later Aristotle adopted a pluralistic, multi-perspectival constructivist view (properties of objects are in observers), not unlike Maturana's (1988: §5.3) "multiverse" or "multiple realities" (Bunnell 2004).

« 8 » Processes and their products are not the same, and should not be conflated. In §30, Totaro and Ninno state,

> "[W]e must necessarily treat numbers as data of observations and not as currently lived processes, since the currently lived process is, by definition, unobservable. A 'thing' is the argument of an observation. Therefore, any observed number is also the argument of an observation, and therefore, a 'thing.'"

Note that two senses of "things" are being used interchangeably, the objects of observations, whether sensuously distinguished tokens or self-observed ideas, and their mental/experiential consequences (products). The assertion that "numbers are observations" (§29) conflates the outcomes of observations (distinctions made by observers) with the process of observing itself, i.e., of making the distinctions. In my view (Cariani 2001, 2022, 2023), numbers are different outcomes of "measurement" operations, i.e., distinctions in the observer, not the measurement process itself. Distinctions and the processes that produce them, although intimately related, are, nevertheless, different, complementary aspects of observers that should not be merged or confused.

Problems within Maturanian phenomenology

« 9 » The proposed analogy attempts to compare separate domains that have fundamentally incompatible metaphysics. I focus here on Maturana's views (Maturana 1988, 2005; Mingers 1991; Imoto 2011). Save perhaps for recursion, number theory does not meld easily with any of Maturana's three ontologies, epistemologies, and phenomenologies. Several explanatory frameworks were adopted by Maturana:

- an Aristotelian hylomorphic working ontology for theoretical biology and neuroscience (Imoto 2011);
- a pluralistic, near-constructivist (Proulx 2008), working epistemology of observers, and
- a highly human-centered, social-constructivist working phenomenology (Maturana 1988, 2005).

When describing subjective experience, Maturana adopts a continental, Heideggerian-like phenomenology (Mingers 1991) that asserts conscious awareness to be the product of human self-referential and self-observational processes engaged in social communications ("languaging"). This is where I part company with him. His extreme human exceptionalism (only humans have consciousness) rules out naturalistic, comparative, biology- and neuroscience-based approaches to the study of both observers and consciousness.

« 10 » In contrast, neurophenomenological approaches seek to identify causal linkages between neural activity and experiential states (Boring 1942; Varela 1996). Maturana's

phenomenology is disconnected from both. Contrary to Totaro and Ninno's assertions in §30, "currently lived" experience *is* introspectively observable by each of us. We do have some conscious access to the contents of our awareness.

Is infinite self-reference possible?

« 11 » My difficulty is not with self-observation *per se*, but with platonic, infinite regressions of such processes, which were explored in depth in Varela (1979). As humans and other organisms, our capacity for self-observation is quite limited in scope and depth.

« 12 » Whereas recursive processes of self-production are all-important, the relevance and depth of self-reference in biological systems, observers, and consciousness is much more limited. There are serious problems, such as the "vicious infinite regress" (Sebastián 2019), with so-called "higher-order thought" (HOT) theories of consciousness based on self-reference. In contrast to such HOT theories, current theories that involve neural global workspaces (e.g., Dehaene 2014) stem from evidence from neuroscience and psychology and better account for states of awareness and their specific experiential contents. In my theoretical variant of neural global workspaces, awareness depends on a particular organization of neural activity in which sets of mutually reinforcing, temporal-coded neural pulse train signals are actively regenerated, i.e., a network of signal self-productions (Cariani 2000, 2015).

« 13 » "Reference" itself is a loaded word that bears many realist connotations, as in referentialist, correspondence theories of truth. "Self-reference" also has many diverse meanings (Pattee & Rączaszek-Leonardi 2012: 211–213). It is critical to indicate the sense of its use to avoid semantic sleights of hand, inconsistent equivalences, and unintendedly altered meanings. Spurred by this discussion, I have pondered its possible meanings. I take its primary senses that are most relevant here to involve self-description and self-pointing. Self-description involves a *partial* description of a whole of which the self-description is a part (a *complete* self-description of the whole would require an infinite regress of self-descriptions). Pointing is a different form of reference. Self-pointing refers to how something within a system can be accessed (e.g., computer address pointer) without describing what is accessed. Self-pointing is not self-production *per se*, although it may be *part of* a process of "bringing forth" components or distinctions.

Self-production and self-reference

« 14 » At this point one might ask: In what sense, if any, is a self-production system inherently "self-referential"? Self-production ≠ self-reference if "reference" is meant in either of the two senses offered above. Clearly the individual metabolic components in a biological organism neither describe nor point to one another, rather, they physically interact. Self-production can be described in terms of finite, ongoing recursive production processes, but recursion alone is neither self-production nor self-reference. Whereas limited self-production and self-reference are possible in biological systems, arguably the infinite recursions and self-references needed for number theory and Cantorian set theory are not. Is any embodied, biological self-production system capable of infinite recursion and self-reference? Q3

« 15 » To summarize, the main dispute here involves a clash between a platonic ontology, in which forms alone are thought sufficient for explanation, and a hylomorphic view in which particular organizations must be embodied in bounded, temporal, finite-capacity, concrete substrates in order to implement their corresponding functions (life, cognition, observation, consciousness).

References

Boring E. G. (1942) Sensation and perception in the history of experimental psychology. Appleton-Century-Crofts, New York.

Bunnell P. (2004) Reflections on the ontology of observing. Cybernetics & Human Knowing 11(4): 72–84. ▸ https://cepa.info/3391

Cariani P. (2000) Regenerative process in life and mind. In: Chandler J. L. R. & Van de Vijver G. (eds.) Closure: Emergent organizations and their dynamics. Annals, New York Academy Sciences, New York: 26–34. ▸ https://cepa.info/5032

Cariani P. (2001) Symbols and dynamics in the brain. BioSystems 60(1–3): 59–83. ▸ https://cepa.info/4139

Cariani P. (2010) Onwards and upwards, radical constructivism: A guest commentary. Constructivist Foundations 6(1): 127–132. ▸ https://constructivist.info/6/1/127

Cariani P. (2012) Infinity and the observer: Radical constructivism and the foundations of mathematics. Constructivist Foundations 7(2): 116–125. ▸ https://constructivist.info/7/2/116

Cariani P. (2015) Outline of a cybernetic theory of brain function based on neural timing nets. Kybernetes 44(8/9): 1219–1232.

Cariani P. (2022) Different roles for multiple perspectives and rigorous testing in scientific theories and models: Towards more open, context-appropriate verificationism. Philosophies 7(3): 54.

Cariani P. (2023) Systems semiotics: A transdisciplinary framework for informational relations in natural and artificial systems. In: Bigliari A. (ed.) Open semiotics. Volume 1: Epistemological and conceptual foundations. L'Harmattan, Paris: 81–103.

Cassirer E. (1955) The philosophy of symbolic forms. Three volumes. Yale University Press, New Haven.

Cassirer E. (1956) Determinism and indeterminism in modern physics: Historical and systematic studies of the problem of causality. Yale University Press, New Haven.

Dehaene S. (1997) The number sense: How the mind creates mathematics. Oxford University Press, New York.

Dehaene S. (2014) Consciousness and the brain: Deciphering how the brain codes our thoughts. Viking, New York.

Feyerabend P. (1987) Some observations on Aristotle's theory of mathematics and of the continuum. In: Farewell to reason. Verso, London: 219–246.

Goodman N. (1972) A world of individuals. In: Goodman N. (ed.). Problems and projects. Bobbs-Merrill, Indianapolis: 155–172.

Graham D. W. (1987) Aristotle's two systems. Oxford University Press, New York.

Imoto S. (2011) The logic of Maturana's biology. Constructivist Foundations 6(6): 325–333. ▸ https://constructivist.info/6/3/325

Maturana H. R. (1988) Ontology of observing: The biological foundations of self-consciousness and the physical domain of existence. In: Donaldson R. E. (ed.) Texts in cybernetic theory: An in-depth exploration of the thought of Humberto Maturana, William T. Powers, and Ernst von Glasersfeld. American Society for Cybernetics (ASC), Felton CA: III.1–53. ▸ https://cepa.info/597

Maturana H. R. (2005) The origin and conservation of self-consciousness. Kybernetes 34(1/2): 54–88. ▸ https://cepa.info/702

Mingers J. (1991) The cognitive theories of Maturana and Varela. Systems Practice 4(4): 319–338. ▸ https://cepa.info/2253

Modrak D. K. (1987) Aristotle: The power of perception. University of Chicago, Chicago.

Pattee H. H. & Rączaszek-Leonardi J. (2012) Laws, language and life: Howard Pattee's classic papers on the physics of symbols. Springer, Dordrecht.

Proulx J. (2008) Some differences between Maturana and Varela's theory of cognition and constructivism. Complicity: An International Journal of Complexity and Education 5(1): 11–26. ▸ https://cepa.info/4338

Rosen R. (1991) Life itself. Columbia University Press, New York.

Sebastián M. Á. (2019) Drop it like it's HOT: A vicious regress for higher-order thought theories. Philosophical Studies 176: 1563–1572.

Varela F. J. (1979) Principles of biological autonomy. North Holland, New York.

Varela F. J. (1996) Neurophenomenology: A Methodological remedy to the hard problem. Journal of Consciousness Studies 3(4): 330–349. ▸ https://cepa.info/1893

Weyl H. (1949) Philosophy of mathematics and natural science. Princeton University Press, Princeton.

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About Origins, Languaging and the Sociohistorical Dimension of Mathematics

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> Abstract • I explore how languaging and explanatory principles are key to making sense of our experiences. Totaro and Ninno's claims about the cognitive origins of continuity and discreteness are examined through a sociohistorical lens. I also emphasize the need for a "poetic" rather than causal understanding of the relationship between our biology and how we conceive mathematical concepts.

Handling Editor • Alexander Riegler

« 1 » In their target article, Paolo Totaro and Domenico Ninno argue in favor of a biological origin of the experience of the continuum and the discrete (§5), which can, for example, be observed in mathematics with natural and real numbers (§17). Rooting the notion of the continuum in the constant (even recursive) autopoiesis of living organisms maintaining their structure, and the discrete in the observation of (momentary) states they call "results," the authors also conceptualize the continuum and the discrete as mutually dependent: "there cannot be a continuum without the discrete" (§42), and vice versa.

Languaging

« 2 » The general idea is reminiscent of Humberto Maturana's (1995) analysis of the concept of time, though with nuances (and perhaps a twist) that seem to be missing from the main article. Totaro and Ninno opt for a symbolic treatment of numbers and the observer (for example §13 and §17), rather than being evocative, redolent or even "poetic" as Maturana often is, for example, when he writes: "we become aware that what we indeed do as we explain our experiences is to use our experiences to explain our experiences" (Maturana 1995: 2). A poetic orientation implies turning our attention to how we make sense of our experiences and construct meaning, with the idea that language is the way in which we bring forth our realities (what I would call its *creative* power), while its "poetic character" allows for richness and flexibility. We could start with questions such as "What do we do when we talk about [the continuum, the discrete, or numbers and so on]?" This approach would conceptualize both the role of the observer and the role of languaging. The continuum and the discrete are primarily acts of languaging, distinctions we make as forms of mutual coordination through which we (literally) *make* sense of our experiences. An argument similar to the one in the target article could be as follows: although we only experience the present moment and current state of ourselves and our surroundings, that experience keeps repeating itself with unceasing variations. We make sense of this experience by distinguishing the "continuum," which concerns the uninterrupted nature of the experience. Additionally, we distinguish in language (and as a manner of coordinating with one another and our surroundings) something we call the "discrete," which concerns what appears to be different (consecutive) states. When we look at people exiting a room, there is a continuous flow of observations, but we can describe the experience as one, then two, then three people exiting the room. These "states" are not fixed in themselves but rather in the account we provide. It is our ability to produce a description, i.e., use language, that makes this apparent fixing possible.

« 3 » A notable illustration of this is Zeno's paradox (formulated centuries before Georg Cantor's conundrum): how can an arrow reach a target if it must first travel half the distance, and how can the arrow get to that half mark if it must first travel half the distance to it, then half the distance to the half mark, and so on? The notions of continuity and recursivity are also constituted in language: it is a way to make sense, organize, and explain our observations, our experiences. When we say that the arrow continuously flies and eventually reaches the target, we constitute it as an event in and through language. When we ask how it can do this when it has to travel recursively (and infinitely) shorter distances, this is another creative act of language, one in which we look for an explanation, what Maturana (2013: 302) calls "an act of poetic synthesis as an