Dewey J. (1960) The quest for certainty. Capricorn, New York. Originally published 1929.

Gash H. (1974) The constructivist epistemology in John Dewey, Jean Piaget, and Cognitive developmental psychology. In: Smock C. D. & Glasersfeld E. von (eds.) Epistemology and education. Follow Through Publications, Athens GA: 27–44. ▸ https://cepa.info/2181

Gash H. & Chocholouskova Z. (2013) A model for promoting constructivist education. In: Lasker G. E., Hiwaki K. & Aydin A. (eds.) Life-long integral education for a viable human future. International Institute for Advanced Studies, Tecumseh Ontario: 39–43. ▸ https://cepa.info/2185

Glasersfeld E. von (1972) Semantic analysis of verbs in terms of conceptual situations. Linguistics 94: 90–107. ▸ https://cepa.info/1312

Glasersfeld E. von (1974) Piaget and the radical constructivist epistemology. In: Smock C. D. & Glasersfeld E. von (eds.) Epistemology and education. Follow Through Publications, Athens GA: 1–24. Available at ▸ https://cepa.info/1324

Glasersfeld E. von (2007) Aspects of constructivism: Vico, Berkeley, Piaget. In: Glasersfeld E. von Key works in radical constructivism. Edited by Marie Larochelle. Sense, Rotterdam: 91–99. ▸ https://cepa.info/1569

Kenny V. (2007) Distinguishing Ernst von Glasersfeld's radical constructivism from Humberto Maturana's "radical realism." Constructivist Foundations 2(2–3): 58–64. ▸ https://constructivist.info/2/2-3/058

Maturana H. R. (1988) Reality: The search for objectivity or the quest for a compelling argument. Irish Journal of Psychology 9(1): 25–82. Available at ▸ https://cepa.info/598

Peirce C. S. (1923) How to make our ideas clear. In: Cohen M. R. (ed.) Chance, love and logic. Harcourt, Brace & Co, New York. Originally published in 1888.

Piaget J. (1970) Piaget's theory. In: Mussen P. H. (ed.) Carmichael's manual of child psychology. Wiley, New York: 703–732.

Steffe L. P., Glasersfeld E. von, Richards J. & Cobb P. (1983) Children's counting types: Philosophy, theory and application. Praeger, New York.

Tourmen C. (2016) With or beyond Piaget: A dialogue between new probabilistic models of learning and the theories of Jean Piaget. Human Development 59: 4–25. ▸ https://cepa.info/6640

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Diving Deep into the Ocean Through Skillful Problem Posing | Solving Experiences

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> Abstract • In this commentary, I extend Videla, Veloz and Pino's conceptual analysis of their design-engineering students' flow of actions by drawing similarities to some of the recent developments in problem-posing and problem-solving research in mathematics education.

« 1 » In their target article, Ronnie Videla, Tomas Veloz and María Carolina Pino demonstrate how the 4E approach to cognition can be used to explain creative experiences of a group of students in a STEAM educational setting involving solving lived environmental problems through design, prototyping, and collaboration. Presenting creativity and problem solving as important skills for the 21st century, Videla, Veloz and Pino also illustrate how creativity can be fostered in STEAM environments that are rich in sensorimotor engagement opportunities, and involve building new artifacts, experience with different types of materiality, and problem-solving experiences that are based on the 4E approach to cognition.

« 2 » Reading the target article made me immediately think of similar conceptual analyses of students' flow of actions, explained from enactivist perspectives, by Jerôme Proulx, Jean-François Maheux, and Robyn Gandell (Proulx & Maheux 2017; Gandell & Maheux 2019) in the field of mathematics education. In their analysis of mathematical problem solving, Proulx and Maheux (2017) ground some of the ideas of the mathematical problem-posing and problem-solving literature in Francisco Varela's problem-posing perspective (Varela, Thompson & Rosch 1991; Varela 1996). Although I have also been conducting educational research with colleagues for quite some time on the co-evolution of problem posing and problem solving, but from a radical constructivist perspective (Sevim & Cifarelli 2013; Cifarelli & Sevim 2014; Cifarelli & Sevim 2015), I am delighted that Proulx and Maheux (2017) offered equally useful explanations that detailed the dynamic interactions between problem posing and problem solving. In their re-conceptualization, Proulx and Maheux (2017) offered *posing | solving*, or *problematizing*, as a continuous dialectical process of small problem-posing and problem-solving instances (Gandell & Maheux 2019), where posing and solving emerge as inseparable activities. As a form of asymmetrical organism–environment coupling (Harvey 2017), posing | solving orients me to consider Videla, Veloz and Pino's students' flow of actions, in which they observe-document-do-reflect-show (§§30–38), as part of project-based learning in STEAM education, as having close resemblance to Gandell and Maheux's (2019) students' posing | solving paths.

« 3 » In posing | solving, there are no problems pre-existing out there, independently of solvers' interactions with them, with built-in conceptual structures or meanings; instead, solvers bring them forth through their interactions with what Proulx and Maheux (2017) and Elaine Simmt (2000) call a "given prompt." When faced with this given prompt, the solver reacts to it and makes it her own by posing a problem. This posed problem, in return, becomes a new prompt (a self-generated prompt) to react to (Sevim 2017). Interactions with, or reactions to, prompts and problems are called problem solving, which bring the solver closer to the solution of the given prompt. The posed problem, as a new environment co-constituted with the solver, brings forth new opportunities for action that are evident here and now, the enactments of which constitute problem solving. These problem-solving movements, either deliberate or spontaneous, result in the solver posing further problems. Through these continuous posing|solving moves, the solver lays down a path (Proulx & Maheux 2017), which Tim Ingold (2010) calls a "line of becoming." Varela (1996) also reminds us of the importance of posing relevant questions that emerge at each moment of our lives, as these questions shape our interactions with a world that we create, which in return creates us.

« 4 » In a sense, posing|solving, as a "making" (Ingold 2010), is akin to what Videla, Veloz and Pino call a constant flow of skillful activity (creativity) that "makes relevant particular characteristics, which tends to reduce the complexity of the activity" (§13). Consistent with Varela's (1996) views on problem posing, this constant flow can be seen as mutual specification and co-determination (coupling) of an organism (a posing|solving) and its environment (also a posing|solving), which continually illuminate opaque facets previously invisible (§3). Thus, from the 4E perspective, how are creativity and problem solving related to each other? Q1

« 5 » Building on Ingold's (2010) work on making, Videla, Veloz and Pino invite us to think of creativity as "a skillful experience embedded in context and arising from the distributed engagement of perception" (§3), and not as a final product or an internal mental process. Similar to creativity, posing|solving is also embedded in context and arises from the distributed engagement of perception and action. Posing|solving:

- moves the focus to students' emergent actions and lived experiences, away from pre-existing concepts and solution strategies (Gandell & Maheux 2019),
- orients our attention toward students' problems that emerge while interacting with given prompts (Proulx & Maheux 2017; Sevim 2017; Sevim 2019),
- is antithetical to the hylomorphic model of creation (Ingold 2010),
- can be viewed as a complex, enmeshed and wayfaring journey (Gandell & Maheux 2019; Ingold 2010),
- can be viewed as a form of enaction that is context specific (Proulx & Maheux 2017; Harvey 2017),
- is a variety of enaction characteristic of a particular form of life (Harvey 2017),
- does not focus on processes involving intentions or mental representations (Proulx & Maheux 2017),
- exemplifies how the brain-body-world interaction links these three parts into an autonomous, self-regulating single system (Carney 2020).

« 6 » This list shows that posing|solving can be seen as a way of creation, which brings forth continuous changes in perception and action that trigger contingent relationships, which, in turn, advance students' level of engagement and reorganize their sensorimotor structures (§§38–40). Perhaps, as these posings|solvings get closer to the optimal solution of the given prompt, we can say the student is engaging in a skillful practice (§38), which the authors call creativity. Thus, to what extent might posing|solving be useful to further our understanding of not only problem solving but also creativity? Q2

« 7 » Let me illustrate posing|solving with an example. I sometimes prepare my students, who are pre-service teachers, for the Praxis Core Mathematics exam. This test measures their core mathematical knowledge for certification purposes in the United States, and it includes questions like the following: A box of machine parts contains 5 times as many usable parts as defective parts. In a box containing 168 machine parts, how many usable parts are there?

« 8 » If I ask my students, "Do you understand the problem?," most of them would say "Yes, I need to find out how many usable parts are in the box." However, this is not the "problem" that I see when I encounter this prompt. The number of usable parts in the given box is the unknown quantity that I need to find, yes, but for me the problem is finding five sixths of 168. While unproblematic to me, the answer to the question of how to find this unknown quantity is neither evident nor immediately accessible to most of my students. Many iterations of posing|solving are often necessary for them to reach an answer.

« 9 » In other words, the whole-to-part structure of a 6:5 ratio, which immediately emerges for me, is not an opportunity for action for most of my students. I have seen students respond to the prompt by trying to generate pairs of numbers where one number is five times the other. They may start guessing that there are 100 usable and 20 defective parts. But 100 and 20 do not add up to 168, so another pair must be considered. Then they may guess that there are 150 usable and 30 defective parts. But 150 and 30 do not add up to 168 either, so, yet another pair must be considered. Many students get stuck here, as this problem-solving activity is not efficient. Note that in this problem-solving activity, the given prompt triggers the solver to pose the problem of finding a pair of numbers that work. I have also seen students writing equations $5U = D$ or $5D = U$ in response to the given prompt. In this posing|solving, instead of finding a pair of numbers that satisfy the given requirements, the posed problem becomes: how to model the given relationship between the two types of machine parts? After some symbol manipulation, another problem might be posed, such as: what to do with the equation $5U = D$? Or, how is the equation $5U = D$ related to 168?

« 10 » To me, the students reported on in the target article (§§29–41) also experience posing|solving. The first prototype they generate is their reaction to the given prompt, and by working on this prototype (posed problem) they generate progressively more effective prototypes that make opaque aspects of the previous prototypes clearer to the present (§6). At this juncture, I would like to draw attention to the role of prompts in both posing|solving and creativity. This is important for our 4E analyses of problem solving and creativity because, as Matthew Harvey (2017) argues, just as sea turtles display geomagnetic-field-sensitive pathfinding and seasonal migration (Cain et al. 2005), problem solvers display prompt-sensitive posing|solving. As sensorimotor environments, prompts are the available contingent relationships that shape the emergent student interactions. While in the math example above, the opportunities for action generated by the given prompt are not endless, so it does not resemble a deep ocean as in David Lynch's (2006) metaphor, the prompt given to Videla, Veloz and Pino's students is rich with sheer opportunities for perception and action.

« 11 » The design-engineering students in the target article collectively design buoy prototypes that can hold a remote sensor to measure fresh-water quality in southern Chile (§§31–41). This problem-solving activity involves open-ended prompts that have multiple strategies, multiple solutions and multiple correct answers, which allow seemingly endless opportunities to "dive deep" and catch ever bigger fish. Moreover, the abundance of sensorimotor engagement, the availability of technological resources, the continued interactions with different types of materiality, and collaboration and discussion with peers within the given STEAM environment, help Videla, Veloz and Pino's students to dive deeper and deeper. They move as they relate to artifacts and other people (§11), and along the way problems are brought forth and solved. Thus, the richer the prompt, the more opportunities for skillful experience, posing|solving, and learning.

« 12 » Finally, I agree with the authors that the traditional view of creativity as having an original concept is not useful for educators who are trying to promote its development, because it is difficult to prepare interventions that can somehow bring about original concepts in learners' minds. By contrast, a more dynamic, embodied, enacted, embedded and extended view of creativity, as skillful experience involving sensorimotor engagement, better equips educators with effective pedagogical strategies such as collaboration, learning by doing, the use of technology and dialogue. We cannot catch the big fish unless we engage in such complex, deep and multilayered engagement with presented prompts and posed problems.

References

Cain S. D., Boles L. C., Wang J. H. & Lohmann K. J. (2005) Magnetic orientation and navigation in marine turtles, lobsters, and molluscs: Concepts and conundrums. integrative and Comparative Biology 45(3): 539–546.

Carney J. (2020) Thinking avant la lettre: A review of 4E cognition. Evolutionary Studies in Imaginative Culture 4(1): 77–90. ▸ https://cepa.info/7884

Cifarelli V. V. & Sevim V. (2014) Examining the role of re-presentation in mathematical problem solving: An application of Ernst von Glasersfeld's conceptual analysis. Constructivist Foundations 9(3): 360–369. ▸ http://constructivist.info/9/3/360

Cifarelli V. V. & Sevim V. (2015) Problem posing as re-formulation and sense-making within problem solving. In: Singer F. M. & Ellerton N. & Cai J. (eds.) Mathematical problem posing: From research to effective practice. Springer, New York: 177–194.

Gandell R. & Maheux J.-F. (2019) Problematizing: The lived journey of a group of students doing mathematics. Constructivist Foundations 15(1): 50–60. ▸ https://constructivist.info/15/1/050

Harvey M. I. (2017) "Posing|solving" can be explained without representations, because it is a form of perception-action. Constructivist Foundations 13(1): 169–171. ▸ http://constructivist.info/13/1/169

Ingold T. (2010) The textility of making. Cambridge Journal of Economics 34(1): 91–102.

Lynch D. (2006) Catching the big fish: Meditation, consciousness, and creativity. Penguin, London.

Proulx J. & Maheux J.-F. (2017) From problem-solving to problem-posing, and from strategies to laying down a path in solving – Taking on Varela's ideas for mathematics education research. Constructivist Foundations 13(1): 160–167. ▸ https://constructivist.info/13/1/160

Sevim V. & Cifarelli V. V. (2013) The co-evolution of problem posing and problem solving in the course of Sarah's on-going solution activity. In: Lindmeier A. M. & Heinze A. (eds.) Proceedings of the annual meeting of the international group for the psychology of mathematics education. Volume 5. PME, Kiel: 165.

Sevim V. (2017) Co-evolution of problem posing and problem solving after finding a way in. Constructivist Foundations 13(1): 173–175. ▸ https://constructivist.info/13/1/173

Sevim V. (2019) Interacting with other people's boundaries, remainders, and static enclosures. Constructivist Foundations 15(1): 67–69. ▸ https://constructivist.info/15/1/067

Simmt E. (2000) Mathematics knowing in action: A fully embodied interpretation. PhD dissertation. University of Alberta, Edmonton.

Varela F. J., Thompson E. & Rosch E. (1991) The embodied mind. MIT Press, Cambridge MA.

Varela F. J. (1996) Invitation aux sciences cognitives. Translated by Pierre Lavoie. Éditions du Seuil, Paris. A shorter English version was published as: Varela F. J. (1992) Whence perceptual meaning? A cartography of current ideas. In: Varela F. J. & Dupuy J. P. (eds.) Understanding origins: Contemporary ideas on the origin of life, mind and society. Kluwer, Boston: 235–265. ▸ http://cepa.info/2074

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