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Varied Repetition in Embodied Learning of Mathematics

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> Abstract • Discussing the ideas of Bernstein, Shvarts, and Abrahamson, I suggest considering the more prominent role of Bernstein's "repetition without repetition" principle in embodied learning. A vignette from an empirical study of co-construction activity illustrates the suggestion.

« 1 » When reading the target article by Anna Shvarts and Dor Abrahamson, I recalled a well-known quote from Kurt Lewin (1951: 169): "[T]here is nothing so practical as a good theory." As a mathematics education design-based researcher conducting studies on learning and teaching in various embodied environments, I am interested in the connection between aspects of mathematical activity related to students' interaction with the material environment and aspects related to mathematical formalism. Thus, bridging the ontological gap between sensorimotor and higher-order processes, which Shvarts and Abrahamson indicate as an impediment to the theory of embodied learning (§2), may facilitate overcoming a practical design challenge for learning where physical actions inevitably lead students to construct mathematical ideas.

« 2 » In order to bridge this theoretical gap, Shvarts and Abrahamson aim to present a "unified theoretical account that reconciles radical embodied approaches to cognition and cultural–historical ideas on higher-order functions" (§12). This account is rooted in the authors' interpretation of and bridging between the work of Nikolai Bernstein and Lev Vygotsky – compatriots, colleagues, and collaborators[1] who worked in Soviet Russia in the 1920s (§9). The authors connect Bernstein's ideas of multilevel self-organization of an organism's action and Vygotsky's cultural-historical approach,

1| At least in one textbook (Artemov et al. 1927) they both are credited as contributors.

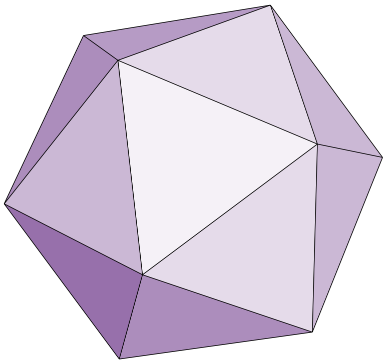

Figure 1 • The icosahedron.

including semiotic mediation and student–teacher collaboration (§15), also mobilizing the ideas from Pyotr Anokhin's functional dynamic systems theory (§14).

« 3 » To demonstrate ontological continuity between motor and mathematical skills, the authors provided an account of a student solving a trigonometric equation $\sin a = \sin 2a$ while using a specifically designed technological tool (§37). By physically changing the height of one point along the Y axes of the unit circle, the student simultaneously manipulated the magnitudes of two angles and the corresponding magnitude of the sine function, which is equal for both. The problem was solved through the tutor's actions, restructuring the student's perception (§41) in concurrence with Shvarts and Abrahamson's treatment of a semiotic activity as a "*physical transformation of the cultural–material environment for the other*" (§44, emphasis in the original).

« 4 » However, a notion central to Bernstein's work seems to be missing from the provided account – "repetition without repetition" (Bernstein 1996: 204). Per Bernstein, through repetition, a learner of a new motor skill repeatedly solves a given motor problem. While doing so, she changes and improves the means for the solution and does not just reiterate them:

> "Repetitions of a movement or action are necessary in order to solve a motor problem many times (better and better) and to find the best ways of solving it. Repetitive solutions of a problem are also necessary because, in natural conditions, external conditions never repeat themselves and the course of the movement is never ideally reproduced. Consequently, it is necessary to gain experience relevant to all various modifications of a task, primarily, to all the impressions that underlie the sensory corrections of a movement." (ibid: 176)

« 5 » Bernstein discerned two phases in learning the motor skill. Repetition plays an important yet different role in both phases. First, "[a] human starts learning a movement because he cannot do it" (Bernstein 1996: 204). Thus, when skill is not obtained, repeated efforts are explorational and are directed to establish some movement pattern leading to a goal. Second, Bernstein suggested that "secrets (of movement) are impossible to teach by demonstration" (ibid: 187), and after establishing a movement pattern, varying conditions leading to various experiences should be purposefully created (Newell 1996). Note a similarity of this idea from learning motor skills to the central tenets of the variation theory of learning (VTS) (Marton & Booth 2013). The VTS conceptualizes learning as a qualitative shift in perceiving the object of learning through experiencing difference (variation) between contrasting values, which leads to the separation of the value from the object of learning, and establishes a dimension of variation.

« 6 » In §14, Shvarts and Abrahamson point out that the ecological environment and actions within have a repetitive quality. This observation aligns with Dragan Trninic's (2018) work, which examined the teaching methods from the physical disciplines, particularly martial arts, and provided a conceptualization of repetitive practice, also in mathematics education, as an exploratory activity integrating properties of direct instruction and discovery learning. Still, the authors came close to using the "repetition without repetition" principle in their analysis of embodied learning, yet fell short. Thus, I wonder: How does Bernstein's principle of "repetition without repetition" manifest itself in the empirical example presented in the target article? Q1

« 7 » Reading the target article made me rethink some of the data I am currently analyzing. Drawing on the authors' theoretical framework, I will extend Shvarts and Abrahamson's argument to reflect on Bernstein's "repetition without repetition" principle and illustrate this with an example from a mathematical embodied activity.

« 8 » In the ongoing research project (Benally et al. 2022; Palatnik 2022), students are given a 2D diagram and written instructions to construct an icosahedron – a polyhedron composed of twenty equilateral triangular faces. The instructions read: "Your team has to construct a model of the 3D solid polyhedron on the figure. The polyhedron has the following properties: All the faces are equilateral triangles, and the same number of edges meet at each vertex." One of the mathematical ideas we want students to grasp is that exactly five edges converge at each vertex of the icosahedron. Being a construction problem, the task at the core of the activity has characteristics of both motor and mathematical problems. Each resource (a construction kit, a diagram, and printed instructions) can be considered a complementary means for the semiotic mediation of mathematical ideas necessary for learning about platonic solids. At some stages of construction, students' actions resulted in model vertices with four and six edges. Thus, the mathematical idea that exactly five edges meet at each vertex of the platonic icosahedron was only partially mediated by semiotic means provided at the beginning of the task. The visibility of one vertex with five converging edges on the diagram (Figure 1) and the corresponding text was insufficient, at least for some participants.

« 9 » In order to advance the problem solution and repair the model, the participants conducted the following actions. They repeatedly read aloud the instructions, with an emphasis on the second parameter (number of edges); repeatedly inspected a 2D diagram (probably with attention shifted to the number of edges); repeatedly re-voiced the incorrect (four) and correct (five) number of edges converging at the vertex; and repeatedly checked the "fiveness" by counting edges (directly touching or pointing to them) on the emerging model (Figure 2). These repeating efforts can be considered the first – exploratory phase of motor-mathematical learning, where "repetition without repetition" occurs. Thus,

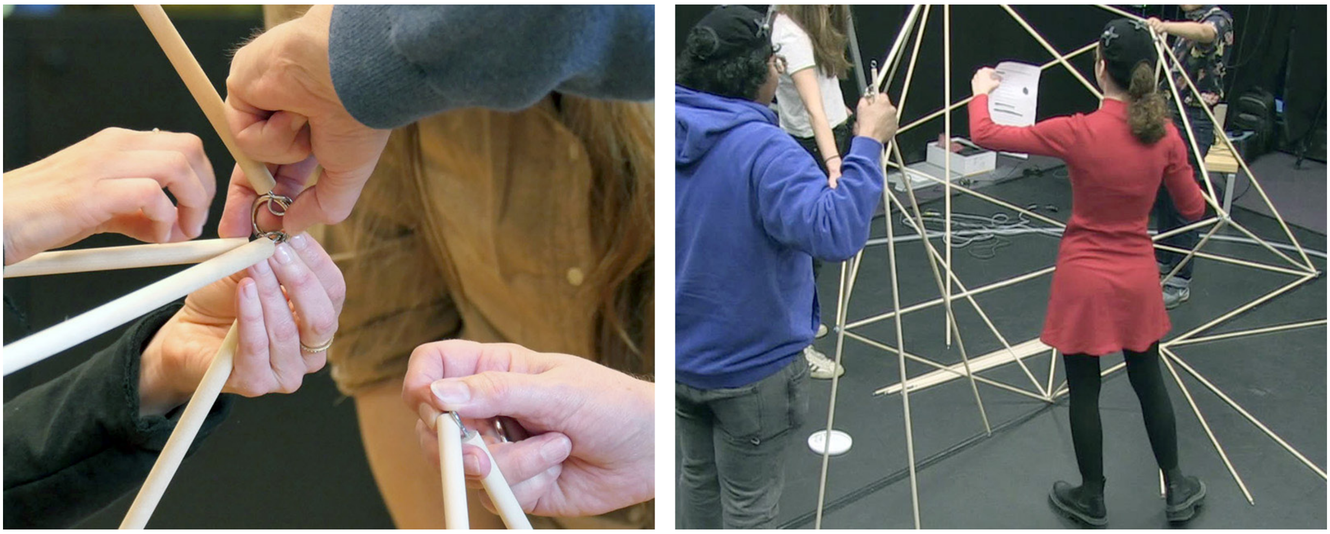

Figure 2 • Elements of the construction kit, and the process. (Left photo was taken by Alexander R. Jensenius).

to construct the model, participants strive for a *better functional grip* on personal and collective levels (§44). They must educate one another's perception (Goldstone et al. 2010) to achieve this improved functionality. Perception is educated through the participants' and model's movement in the environment, resulting in the collection and exchange of multiple visual, kinaesthetic, and tactile perspectives on the emerging model.

« 10 » The emerging model is an artifact with a special part in this complex functional dynamic system. As students construct the model, they construct their knowledge about a particular polyhedron and improve their skills in solving co-construction problems. This is not a mere metaphor. At the more advanced stages of construction, the students worked with, perceived, and named complex sub-structures of the model ("star," "base," "pentagon"), which hierarchically included more simple sub-structures – "triangular faces." (These substructures can be found on the diagram and on the photo of the activity). At the later stages of the co-construction activity, the students utilized these substructures to complete the construction successfully and discover several properties of the icosahedron: symmetries, number of edges, faces, and vertices. In other instantiations, the students successfully constructed the same model on a different scale (Palatnik 2022). These more advanced efforts, in which students operate with more complex structures in a varying environment (for instance, by changing the position of the whole model in space), can be considered the second phase of motor-mathematical learning. In line with Bernstein, at this phase, "repetition without repetition" facilitates the discovery of properties that cannot be taught by demonstration alone or perceived through passive observation.

« 11 » Concluding the target article, Shvarts and Abrahamson suggest several tenets for theoretical reconsideration of semiotic mediation in embodied learning of mathematics (§44). In my commentary, I have argued for an additional consideration – varied repetition. The learning ecosystem should include carefully and purposefully varied opportunities for repeated problem solving involving sensorimotor and mathematical aspects. Learners' repetitions of collaborative attempts to solve these problems may be connected by semiotic means, which, in turn, can be transformed to allow better perception-guided action in the problematic situation.

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Could Education in the 21st Century Embrace Fuzziness, Ambiguity, and So On?

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> Abstract • While the target article articulates key concepts from enactivism and social perspectives very well, the framework the authors offer seems to obscure some epistemological tensions. One such tension concerns the goal of research and how the framework can contribute to education in the 21st century. The second is about the way in which the authors conceptualize the relationship between the theorizations they draw on. The third is in the adoption, at times, of a reifying perspective in contradiction with the "dynamical" approach the authors first adopt. All three can be seen as opportunities to embrace and promote fuzziness and ambiguity for 21st-century (mathematics) education.

« 1 » Anna Shvarts and Dor Abrahamson offer a target article in which they connect Lev Vygotsky's and Nikolai Bernstein's work in a coherent fashion. An enactivist perspective helps us understand how Bernstein's observations and conceptualization of bodily movement dynamics could hinge on (and perhaps support?) Vygotsky's higher cognitive functions through, for example, the use of artifacts. The authors also briefly illustrate the possibilities of this united framework for talking about mathematics education phenomena. More precisely, they draw on eye-tracking technology to examine how a student and a tutor work together on a simple mathematical problem.

« 2 » I deeply appreciate the idea of reconceptualizing "semiotic mediation" from an enactivist perspective as a direct social extension of bodily dynamics. While the authors often do a very good job of articulating key concepts from enactivism and social perspectives, some epistemological tensions seem to be overlooked. For one, I feel that a broader framework to situate how they conceptualize research and this particular endeavour is necessary. Explicitly presenting such a framework might also be an opportunity to explain how the authors see their work in regard to the theme of this special issue. And this makes me wonder: How does a "a functional dynamic systems perspective" connect with "Education in the 21st Century"? Q1 My intuition and understanding are that the authors should not (however tempting this may be) argue that this perspective leads to new (or worse: "better") instructional or pedagogical directions (models, activities, etc.), and nor should they claim that it can change the face of mathematics education. For me, observations and interpretations such as the ones they offer illustrate many new pathways for *research*,[1] to develop *new understandings of what is going on* in teaching and learning. Perhaps, and I insist, *perhaps* it can inspire educators in how they work with students.

« 3 » In direct line with these remarks is my second question. On a few occasions, and this is key to their proposal (e.g., §§6, 9), the authors tell us that while works of Vygotsky and Bernstein seem to address different aspects of cognition, they "need to be understood as complementary" and lead to "important insights for a general theory of cognition and for educational design and practice." The question of educational design surely touches upon Q1, which might also be interpreted as: What exactly does this framework contribute in terms of educational design and practice (e.g., for mathematics education), and how is it significant compared with what decades of research (e.g., in mathematics education) have already shown us? However, my second question is about *complementarity*.

« 4 » I wonder, from an epistemological perspective, what it means here to be complementary? Are the authors suggesting that Vygotsky's and Bernstein's work together "completely" explain or address the phe-

1 | I am thinking, for example, about the concerning trend to draw simplistic educational insights from neuroscience data (e.g., brain-scans of a handful of people performing simple mathematical tasks), with catchy inflated labels like "brain-based education," "neuro-didactic," "brain target teaching," and so on. There is room for interdisciplinarity work between educators and neuroscience research, and the work presented here sounds to me like a promising pathway.