Coordination Dynamics of Semiotic Mediation: A Functional Dynamic Systems Perspective on Mathematics Teaching/Learning

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> Context • Radical embodied approaches to cognition propose a drastic alternative to representation-based models of the mind by way of theorizing and empirically demonstrating the constitutive roles of perception–action loops in human behavior. However, applying those approaches to higher-order processes – such as mathematical thinking and learning – remains one of the hottest debates within contemporary cognitive science. **> Problem** • How might a radical embodied perspective theoretically explain semiotic mediation? For example, how can we theorize the understanding of trigonometric relations expressed through the symbolic form of "$\sin 2a = \sin a$"? **> Method** • Revisiting the nature of semiotic mediation, we pursue a historically grounded theoretical analysis that integrates perspectives from Lev Vygotsky and Nikolai Bernstein; to make the theoretical proposal accessible, we illustrate it by empirical data from a dual-eye-tracking study on teaching/learning trigonometry. **> Results** • We show how semiotic mediation of mathematical ideas is constituted as direct intercorporeal sensorimotor coordination between interlocutors. We treat semiotic actions as directly transforming an environment for the interlocutor, inviting new sensorimotor processes. New sensorimotor routines potentially lead to the emergence of pedagogically desired perception and action orientations, that is, new enactive capacity as the cognitive vehicle of mathematical reasoning. As such, Vygotskian cultural–historical ideas of semiotic mediation become a natural continuation of a radical embodied project developed by Bernstein, Kelso, Turvey, and others. Semiotic actions do not represent mind-independent reality as standalone tokens – rather, they present the environment itself, for the interlocutor, in a new way. **> Implications** • The proposed perspective avoids ontologically problematic views of pedagogical discourse as the negotiation of minds, instead focusing educators' attention on transformations of the students' environment that foster desired perceptions and actions. **> Constructivist content** • We develop an alternative to a social-constructivist reading of cultural-historical ideas, thus contributing to the understanding of higher-order cognition as direct extension of perception and action.
> Key words • Bernstein, complex dynamic systems, coordination dynamics, functional systems, intercorporeal systems, mathematics education, multimodal joint attention, semiotic mediation, Vygotsky.

Introduction

« 1 » Scientific understanding of the human mind has been undergoing deep transformations with the rise of the embodied paradigm in the cognitive sciences (Newen, Bruin & Gallagher 2018). In turn, these embodied approaches have been transforming the learning sciences, as evidenced in embodiment theoreticians treating educational issues (Shapiro & Stolz 2019), learning scientists publishing special issues on embodiment and education (Hall & Nemirovsky 2012), and the International Society of the Learning Sciences thematizing embodiment approaches in its annual meeting (CSCL 2019). As educational researchers evaluate tenets and implications of embodiment, a particularly bold proposal comes from radical embodied approaches that deny the representationist nature of cognition (e.g., Chemero 2009) and re-conceptualize the environment from an ecological perspective (Turvey 2019). These radical reformulations of cognition theory challenge widespread assumptions regarding the ontology of mathematical concepts and the epistemology of mathematics students learning these concepts.

« 2 » Attempts to reformulate educational theories as drawing on embodiment perspectives fall short, as the 4E paradigm – the notion that cognition is embodied, embedded, extended, and enacted – has not yet satisfactorily theorized cognition beyond human–environment sensorimotor coupling (Hutto & Abrahamson 2022). On the one hand, non-representationalist accounts of motor actions and perceptual processes are well grounded in the field of coordination dynamics (Turvey 1977; Kelso & Schöner 1988), which have provided detailed experimental demonstrations of complex-dynamic-systems processes in regulating behavioral change. Those accounts cohere (Baggs & Chemero 2021) with phenomenological and ecological-psychology perspectives on ontology that approach phenomena not from an objectivist definitive perspective but per their ad hoc, contextual, and goal-oriented subjective relevance for interaction (Turvey 2019). On the other hand, radical embodied approaches to higher-order processes are still embryonic (Sanches de Oliveira, Raja & Chemero 2021; van Dijk & Rietveld 2021; Veissière et al. 2020) and polemic (Goldinger et al. 2016; Pulvermüller 2013) and have not yet resolved the persisting explanatory dichotomies between physical and intellectual practices. The ontological gap between sensorimotor processes as functionally entangled with the material environment and higher-order processes that operate mental models impedes a holistic theoretical vision of the embodied mind engaged in mundane socio-cultural activity. As a result, educational researchers inspired by embodied approaches who aim to understand and facilitate mathematics acquisition choose a moderate embodied position when talking about the role of the body in educational design (e.g., Duijzer et al. 2019) or combine embodied and representational perspectives (e.g., Núñez, Edwards & Filipe Matos 1999; Bartolini Bussi & Mariotti 2015).

« 3 » An account provided in this article aims to seamlessly embroider cognitive processes as responsible for both *motor* and *mathematical* practice by theorizing skill – any human skill including semiotic activity – as the functional dynamic organization of perception and action. Experimental educational paradigms are creating new contexts in which to explore the ontological continuity from motor to mathematical skill, whether teaching–learning to flip a pancake or solve an equation for *x*. Our approach to conceptual learning endorses classical cultural–historical ideas on *semiotic mediation,* namely, involvement of words, formulas, visual notations, and other cultural artifacts, as an indispensable feature of higher-order cognition (Vygotsky 1978; Leontiev 1978; Cole 1996; Roth & Radford 2011; Bartolini Bussi & Mariotti 2015). At the same time, our approach also endorses systemic analyses of physical movement performance (Kelso & Schöner 1988; Bernstein 1996; Mechsner et al. 2001; Sheets-Johnstone 2015) within the ecological environment (Gibson 1986; Heft 1989; Turvey 2019). *We propose to re-conceptualize semiotic mediation as a direct social extension of bodily dynamics in cultural ecologies.* We ask: *How can we describe semiotic mediation processes in consonance with the science of coordination dynamics?*

« 4 » In what follows, we will present a monistic approach to organism–environment interaction. This monistic approach proposes a coherent combination of, on the one hand, coordination dynamics' findings on action regulation and, on the other hand, the cultural–historical view on artifacts' mediating role in acquiring higher-order skills (see also Abrahamson 2021; Abrahamson & Trninic 2015). We elaborate this monistic theoretical conciliation as *intercorporeal bodies–artifacts functional dynamic systems* (Shvarts et al. 2021; Shvarts & Abrahamson in press). Our theoretical proposal is meant to resolve an ostensible ontological discontinuity between materiality and mental cognition, specifically looking to explain mathematical cognition as an extension of sensorimotor skills (Abrahamson 2021).

« 5 » We intend the article as a sketch for a possible theoretical vision of mathematical thinking, teaching, and learning phenomena. In the following sections, we provide conceptual historical ground for our ideas through an excursion into the historical origins of coordination dynamics and the cultural–historical approach (Section 2); outline our theoretical proposal (Section 3); and provide a brief illustration from empirical studies on hand and eye movements of a student and a tutor as they collaboratively solve mathematical problems (Section 4). These historical considerations and empirical excerpts are put forth not as providing strict evidence supporting our approach. Rather, they are intended to facilitate our presentation of the main theoretical proposal in Section 3 by providing a context for grasping those ideas.

Historical prelude: Lev Vygotsky and Nikolai Bernstein

« 6 » In early 20th century Russia, two researchers developed strong theoretical systems that later grounded large fields of studies. Vygotsky developed a cultural–historical approach to psychological functions (Vygotsky 1978), aiming to uncover the uniqueness of human cognition based on the historical and cultural development of the human species. His core idea of semiotic mediation and student–teacher collaboration as forming higher psychological functions grounded a vast range of cultural–historical studies in education, particularly in mathematics education (e.g., Radford & Roth 2011; Bartolini Bussi & Mariotti 2015; Cole 2016). Bernstein created a novel (and now widely accepted) methodology of recording movement dynamics, which led him to theorize the development of movement skills (Bernstein 1967). His physiological theory of movement construction kindled the development of the systemic field of coordination dynamics (e.g., Turvey 1977; Kelso & Schöner 1988). At first blush, the respective works of Vygotsky and Bernstein seem to address different aspects of cognition. However, we claim that these systems need to be understood as complementary and that their joint elaboration leads to important insights for a general theory of cognition and for educational design and practice (Abrahamson & Trninic 2015).

« 7 » Per Vygotsky, higher psychological functions, such as counting or writing, are social in origin; the relation between lower and higher functions is *complex*:

> “Higher mental functions are not built up as a second story over elementary processes, but come as new psychological systems that include a complex merging of elementary functions that will be included in the new system, and themselves begin to act according to new laws.” (Vygotsky 1999: 43)

These new psychological entities are mediated by cultural tools and artifacts, including words; they are also *systemic,* as they coordinate multiple subsystems, such as motor actions, perception, and speech, in fulfilling some culturally relevant function.

« 8 » Bernstein is known less in educational literature, as the field of his studies was movement construction (Bernstein 1967). His contribution lies in a detailed account of motor actions' complex dynamics, which he developed in opposition to a then-dominant behavioristic model based on conditioned and unconditioned reflexes (Pavlov 1927). Bernstein focused on investigating human motor activity – for example, throwing a ball or playing the piano – as solving *motor problems* posed within complex environments. The main concern for Bernstein was the problem of too many degrees of freedom: How does the human cognitive system manage the performance of physical movements involving the coordinated micro-motor action of dozens of bodily muscles? How is this possible given that the environment is never quite the same? Just lifting a coffee mug – if controlled muscle-by-muscle as a system of direct responses – becomes an impossible task! Bernstein (1967) theorizes a complex and dynamic system comprising multiple *levels* – such as tackling gravity or orienting in space – that become coordinated in fulfilling a motor problem at hand. Bernstein's multilevel model of action is best known for the systemic idea of *synergies* (Kelso & Schöner 1988): higher (psychological) levels of action regulation propagate "down" as constraints facilitating self-organization of partially independent lower-level processes into coherent movement ensembles.

« 9 » Vygotsky and Bernstein developed their theories independently, and yet, we argue, these theories are complementary. Both scholars took holistic views on physiology and perception, developed earlier by Kurt Goldstein, Jakob von Uexküll, Kurt Koffka, and others, as fundamental prin¬ciples and sources of inspiration (Vygotsky 1978; Sirotkina & Biryukova 2015: 271). Curiously enough, Vygotsky and Bernstein, both born in 1896, worked together in 1925–27 in the Institute of Psychology, where they assisted each other in their studies (Sirotkina 1996). Whereas their research foci diverged – Vygotsky concerned more with higher cognition, Bernstein more with the organization of bodily motion – already Alexander Luria, their mutual friend and collaborator, considered them as providing a unified theory of the human mind and enactment (Luria 1973: 247).

« 10 » Vygotsky speaks of higher mental processes as "meaningful functional systems," which are to be understood based on the following assumptions:

> "(a) the assumption of plastic, changeable inter-functional relations;
> (b) the assumption of *complex dynamic systems* which have to be considered as the result of integration of elementary functions; and
> (c) the assumption of a categorical reflexion [sic] of reality in the human mind." (Vygotsky 1965: 382, our emphasis)

The first two assumptions are directly in line with Bernsteins's idea that lower biomechanical levels (e.g., maintaining a posture) self-organize to solve a motor problem imposed at a higher level (e.g., throwing a ball). The third assumption departs from investigations of movement per se to treat categorization, the psychological process of recognizing classes of objects. Vygotsky clarifies that the higher cognitive process, such as categorization, relies on "'extra-cerebral' connections" (Vygotsky 1965: 385); thus he locates higher processes as distributed between brain and tools. In the thesis of extra-cerebral localization of cognitive functions, Vygotsky enriches Bernstein's ideas by an assumption that is now called extended cognition (Clark & Chalmers 1998; Kirchhoff & Kiverstein 2019).

« 11 » Bernstein, in turn, while limiting the focus of his analysis to explicit physical movements, hypothesizes an additional level in human action regulation, which he addresses as the *symbolic level* (Bernstein 1967): particularly, this psychological level is involved in writing, speaking, and artistic performance. The symbolic level addresses motor problems requiring higher-level characteristics (e.g., in expressive theatrical or musical performance) and develops its own repertory for anticipating and correcting motor performance.

« 12 » Whereas Vygotsky and Bernstein drew from similar intellectual wells, cultural–historical activity theory and coordination dynamics developed as separate research fields, each bearing unique contributions to educational science. The contribution of Vygotsky's legacy to educational science barely needs introduction. Bernstein's ideas highly impacted key figures in the field of complex dynamic systems (e.g., Turvey 1977; Kelso & Schöner 1988) – direct originators of contemporary radical embodied cognitive science. Indeed, a search conducted on 15 March 2021 in Scopus for papers citing "Vygotsky L. S." and "Bernstein A. N." (in relevant fields) reveals 38,399 and 2,638 quotations for each, respectively, and just 86 papers citing both authors. While the cultural–historical approach has incorporated ideas from embodied cognition (Bartolini Bussi & Mariotti 2015; Radford 2021), by and large it has not embraced coordination dynamics principles (but see, e.g., Tancredi, Abdu et al. 2022). Simultaneously, researchers within radical embodied approaches who have become aware of the fundamental roles of cultural artifacts in higher-order cognition (e.g., Malafouris 2019; Sanches de Oliveira, Raja & Chemero 2021) rarely cite the cultural–historical approach that has been promoting this idea for almost a century. It thus appears timely to develop a unified theoretical account that reconciles radical embodied approaches to cognition and cultural–historical ideas on higher-order functions.

Theoretical development: A functional dynamic systems perspective on semiotic mediation

Ecological ontology of culture

« 13 » The theoretical perspective of this article is based on reconsidering realistic ontology – the objective existence of things independently from a cognizing subject – towards a more subtle onto-epistemological view. In consonance with ecological psychology (Gibson 1986) and with cultural-historical calls for eliminating views of ontology and epistemology as separate presumptions (Stetsenko 2020), we adopt an *ecological ontology*, in which the existence of a quale for an individual is contingent on its relevance for an enactment within an organism–environment system (Gibson 1986; Turvey 2019). This onto-epistemological perspective considers the world as objectively revealing itself in different manners for different forms of life (Rietveld, Denys & van Westen 2018). Leaving behind details of philosophical debates, we use the notion of *environment* as an ecological niche, which presents an organism with affordances – direct opportunities for enactment (Gibson 1986) – instead of cold physical qualities of mind-independent reality. Analysis of cultural environments as replete with artifacts suggests theorizing the environment as a system of *nested affordances* (Rietveld, Denys & van Westen 2018) that enable both naïve (material) and cultural (ideal) forms of enactment[1] (Cole 1996; Vygotsky 2001). For example, a cup holds an affordance for throwing as one would a stone but also for drinking tea. Moreover, a cultural environment includes multimodal semiotic means, such as a companion's exclamation, "Look, this cup can serve as a paperweight!" Yet our approach further complements the assumption of nested affordances in the environment with *brain–body potentialities* of the *organism*, that is, the organism's inherent and acquired counterparts for engaging with affordances (Shvarts et al. 2021). Highlighting the organism's role in exploiting organism–environment affordances becomes instrumental in educational research discourse, where we assume that the organism, i.e., a student, is to develop new potentialities. Then one can ask how these potentialities may develop and what roles other cultural agents may play in fostering this development, so that new affordances in the cultural environment come forth for the student. That is, the student learns.

A functional grip of a dynamic system and multilevel intentionality

« 14 » Scholars of radical embodied cognitive science have been seeking to bridge ideas from ecological psychology toward enactivism and complex dynamic systems theory to highlight cognitive activity as the continuous enactment of a *complex dynamic system* of sensorimotor processes (Baggs & Chemero 2021; Rietveld, Denys & van Westen 2018). Working largely in consonance with these ideas, we bring forth a historical version of the complex-dynamic systems approach that highlighted the *functionality* of its performance. Vygotsky's and Bernstein's ideas were developed synchronously with those of Pyotr Anokhin, whose research program addressed the plasticity of physiological processes (Anokhin 1975). The theory of *functional dynamic systems* highlighted that physiological processes spontaneously and flexibly re-assemble to fulfill a behavioral function (e.g., walking) within a changing environment vis-à-vis available neuronal and muscular resources, thus giving rise to "an adaptive effect in the organism-environment interaction achieved upon realization of that system" (Alexandrov et al. 2018: 2). A functional system bears an evolutionary advantage by exploiting the recurrence quality of the ecological environment and actions: based on previous partially repetitive experiences, the system anticipates how the environment will impact the organism at the completion of action (Anokhin 1962). This anticipatory mechanism has also been conceptualized as a *forward model*, namely anticipatory neural circuits preactivated *ahead* of observable enactment, thus allowing one to attune the action to the environment on the basis of any detected discrepancy between anticipated and received sensations (Bernstein 1967). The future-directedness cognitive mechanism of anticipation can be conceptualized as *striving for a better functional grip* (q.v., a system's drive for relative equilibrium, Merleau-Ponty 2002, or its "tendency towards a grip on multiple affordances," Rietveld, Denys & van Westen 2018: 44).

« 15 » This fundamental idea of *striving for a better functional grip* propagates to semiotic mediation through the notion of *multilevel intentionality*. At the psychological level, behavior is organized as the goal-oriented, intentional activity of solving problems, including *motor problems* (Bernstein 1967, e.g., hammering a nail) or *mathematical problems* (solving a trigonometric equation). Yet, this psychological level is only one of multiple anticipatory levels acting simultaneously within a functional dynamic system.

« 16 » Multiple levels of intentionality – besides conscious goal-orientation – have been theorized as *Ur-intentionality*: at the level of a bacterium's directedness along the chemical gradient (Hutto & Satne 2015); *motor intentionality* that orients a hand as it treats an object or greets a friend (Merleau-Ponty 2002); or *enactive intentionality* (shared motor intentionality), as when football players immediately read other players' directionality without mind-reading (Gallagher & Miyahara 2012). These philosophical ideas are supported by physiological and physical studies showing that a system's *ad hoc* state is best explained by its future functional outcome. In laboratory experiments, response times to a visually presented object are briefer when the response button resembles the shape of the object, as if the organism is ready to grasp the object (e.g., Tucker & Ellis 1998). The retina, a sensory receptor, is pre-activated to facilitate recognition of motion or simple configurations (Souihel & Cessac 2021; Zipora, Shimon & Ehud 2021), and eardrums oscillate in anticipation of visual sensory input to coordinate across modalities (Gruters et al. 2018). Some researchers discern end-directedness even in physical dissipative systems allegedly *seeking* states that increase entropy (Dixon et al. 2015). In a sense, functional dynamic systems are teleological at the level of neurons (Alexandrov et al. 2018) and bodily periphery: an entire body gets pre-activated as it anticipates the sensations of efficient enactment. Overall, a functional dynamic system develops to enhance *a functional grip* on the ecological environment at multiple levels, including sensorimotor and social interactions, by anticipating the environment, as it would appear when enactment is accomplished.

Sensorimotor dynamics and perceptual orientations

« 17 » Taken from a functional dynamic perspective, enactment can be understood as a multi-level phenomenon. *Sensorimotor* processes are activations of receptors and muscles – whether they are in the eyes, ears, fingers, or tongue – and brain structures; they are organized in multiple synergetic levels that are partially independent. *Perception and action* are higher-level emergent synergetic units that are phenomenologically given and, as such, meaningful for psychological and educational analysis.

1 | See more on merging the cultural-historical idea of naïve and ideal forms of action and the idea of nested affordances in an upcoming article currently under review, "Reifying actions into artifacts: An embodied perspective on process-object dialectics in higher-order mathematical thinking" by Anna Shvarts, Rogier Bos, Michiel Doorman and Paul Drijvers.

« 18 » Critically, a possibility for action does not determine top-down the sensorimotor organization of a strictly unique response to environmental stimuli. Rather, functional dynamic systems are inherently contingent and flexible in achieving target outcomes (Bernstein 1967) through self-organizing into relative synergetic stability (Kelso & Schöner 1988). Complex environment–organism interaction systems "exhibit self-organization and emergent processes at multiple levels" (Thompson & Varela 2001: 421), each of them driven by their own intentionality yet impacting other levels, as "emergence involves both upward and downward causation" (ibid).

« 19 » The interplay of systemic levels is exploited by experienced sport educators (Chow et al. 2007; Newell & Ranganathan 2010). Swim coaches teaching the crawl style might tell you to "push the water back," leaving you to figure out how to implement biomechanically that top-down imagistic instruction (Stoate & Wulf 2011; see also Hutto & Sánchez-García 2015). Reciprocally, investigators of problem-solving and mathematical learning have noted the bottom-up organization of sensorimotor activity into a well-formed perceptual structure, as, for example, coordinating speech, gestures, or specific eye-movements may bring about an insight moment or an efficient cognitive strategy (Church & Goldin-Meadow 1986; Grant & Spivey 2003; Stephen et al. 2009). Specifically, mixed-methods analyses of mathematics students' hands-and-eye movements and their verbal–gestural utterances reveal the self-organization of sensorimotor processes into new, mathematically relevant, perceptual orientations known as *attentional anchors* (Tancredi, Abdu et al. 2022). Overall, as a new synergetic invariant of *sensorimotor processes* emerges at the physiological level, at the psychological level of analysis we see a new *form of action* and new *perceptual orientation*, thus bridging sensorimotor processes and direct ecological perception (Maturana & Varela 1992; Heft 2020). While motor synergies are critical for the analysis of coordination dynamics, the analysis of educational practice in mathematics foregrounds *perceptual orientations that emerge as the students' means of organizing the performance of prescribed tasks.*

Cultural artifacts as stabilizers of functional systems' dynamics

« 20 » Within the cultural-historical tradition (Leontiev 1978; Wenger 1998; Radford 2003), artifacts are conceptualized "as embodiments of certain cultural practices, crystallized templates of actions, schematized representations of certain ways of doing things as discovered in the collaborative history of humanity" (Vianna & Stetsenko 2006: 97). Think of a spoon that crystallizes the practice of eating. Appropriated and fashioned by subsequent generations, artifacts expand cognitive activity beyond skull, skin, and the individual, in consonance with extended cognition ideas (Vygotsky 1965; Clark & Chalmers 1998; Di Paolo 2009; Menary & Gillett 2022).

« 21 » As physiological and phenomenological research shows, artifacts change body schema transform perception of reachable space (see Weser & Proffitt 2021, for an overview), allowing one to perceive and act with the world that lies on the other side of scissors or a guide cane (Gibson 1986: 40f; Merleau-Ponty 2002). From a functional dynamic systems approach, the cognitive system is extended when an artifact is appropriated into a functional system. An artifact becomes a direct extension of the body as it is incorporated into perception–action loops (Lockman 2000) per the task at hand (Bril 2015). A spoon extends the hand, thus partaking in a perception–action loop. We suggest calling the incorporation of an artifact into perception–action loops the genesis of *a body-artifacts functional system* that enables instrumented actions (Shvarts et al. 2021): artifacts now come to physically mediate perception and action.

« 22 » Vygotsky viewed cultural artifacts as playing constitutive roles in higher cognitive functions (Vygotsky 1978). Elaborating from embodiment perspectives, we suggest that artifacts, such as mathematical visualizations, partake in perception–action loops just like a cane or a pair of scissors do (Shvarts et al. 2021). For example, perceiving points on a Cartesian plane (Krichevets, Shvarts & Chumachenko 2014) and data on histograms (Boels, Bakker & Drijvers 2019) requires specific sensorimotor processes: educated perceivers move their eyes in a particular manner so that they make use of the diagrams' cultural affordances in reading relevant numeric data. An artifact itself preserves cultural practice – historically emerged stable dynamics – because its form specifies situated enactment as it is shaped within the practice. For example, a violin bow privileges a particular grasp of the hand at the nut, and the beads of a Soroban (Japanese abacus) match the counting hand (Monaghan, Trouche & Borwein 2016).

A functional dynamic system beyond one body

« 23 » In this section we extend principles of functional dynamic systems to situations of social collaboration. When two or more people collaborate on achieving a common goal of joint physical action within a shared space and time, they characteristically manifest tight intercorporeal coordination of sensorimotor processes. Coordination emerges spontaneously as people tap on a desk or swing on rocking chairs together (Oullier et al. 2008; Richardson et al. 2007), and people may achieve movement calibration at the level of milliseconds in dance or other motor performance (e.g., Sebanz, Bekkering & Knoblich 2011; Kimmel & Preuschl 2016). Intercorporeal coordination can be further found as two brains coordinate in wavelength and phases of neuronal activity (e.g., Liu et al. 2016; Fuchs & Kelso 2018) or in heart rhythms (Fusaroli et al. 2016). When people operate collaboratively on a common visual display, the quality of their collaboration is correlated with their dynamic gaze coordination (Schneider et al. 2018). Moreover, coordination increases as collaboration on a joint problem continues (Dale, Kirkham & Richardson 2011). Two bodies may couple their dynamics so tightly that the phenomenon can only be explained by assuming that the individuals are actively anticipating each other's sensorimotor processes, thus manifesting motor intentionality at the social level.

« 24 » Granted, two bodies often reach joint aims without *full* congruency of sensorimotor processes. Yet, in situations such as carrying a table together or solving a puzzle together (Sebanz, Bekkering & Knoblich 2011), intercorporeal processes play a decisive role, where the bodies are coupled around a common piece of the environment. Collaborators jointly attend to the environment and coordinate along one or more sensory modalities – such as looking and touching (Yu & Smith 2016; Shvarts & Abrahamson 2019; Pagnotta, Laland & Coco 2020), so that their perceptions of the environment may become sufficiently congruent to enable joint achievement of a mutually desirable outcome. In so doing, new features or relations within the environment may emerge as relevant for the task and themselves become objects of *multimodal joint attention* ready to be captured semiotically. Perception–action dynamics between people thus stabilize.

« 25 » Collaborating partners engaged in joint action do not build a representation of each other's minds (Gallagher & Miyahara 2012). Consider, for example, how time constraints would prohibit football-team players from doing so while in rapid action (Gallagher 2011). Rather, sensorimotor processes self-organize across the bodies into a system that exhibits *multilevel intentionality* towards a common action outcome: the bodies directly anticipate each other at the sensorimotor level, while collaborators are conscious only of their general action goal. The mirror-neuron theory, by which watching others' actions activates one's own corresponding motor neurons, might explain how *forward models* propagate beyond an individual, thus revealing an embodied level of anticipation within an intercorporeal system. Notably, mirror neurons would orient individuals toward the *outcomes* of each other's action, not towards spatial, morphological, or kinematic details of those motor processes (Rizzolatti et al. 2014). Collaborators directly couple their sensorimotor processes into a single *intercorporeal functional dynamic system* that subsume both bodies anticipating each other at multiple levels as they pursue a common goal (Newman, Griffin & Cole 1989; Shvarts & Abrahamson in press).

« 26 » While dynamic intercorporeal coupling of sensorimotor processes is evident in physical interactions, we propose a view of semiotic activity, too, as a sensorimotor process. To begin with, *speaking* consists of locutionary motor actions generating signals in the audial modality. Audial signals propagate through the environment causing direct coordination between bodies:

> "[T]he air's molecules are made to vibrate following oscillatory patterns that can be controlled by articulators including the vocal cords, the tongue, the different parts of the oral tract, and, occasionally, the nasal cavity." (Bottineau 2010: 272)

In this sense, speaking is remote-touching. Just as joint actors align their bodies in pragmatic actions, so they align their sensorimotor systems in semiotic action: accents, pause patterns, voice intensity, and so on increasingly align among interlocutors while communicating (see Dale et al. 2014). Yet, in order to explain how semiotic activity coordinates people not only at the sensorimotor level but at the level of common understanding, we will now extend the idea of bodies-artifacts functional systems to semiotic situations.

Semiotic activity capturing, preserving, and re-activating perceptual orientation

« 27 » Human capacity to expand collaboration between bodies beyond collocated synchronous moments provides an evolutionary advantage, such as in signaling the presence of a predator or hunting large prey. A theorization of language as extending human collaboration beyond joint action agrees with evolutionary anthropologists' thesis on the phylogenesis of speech (Donald 2010) and cognitive developmental psychologists' thesis on the ontogenesis of speech (O'Madagain & Tomasello 2021). Language, and, more generally, semiotic means, are human artifacts. Similar to other cultural artifacts, semiotic means are crystallized forms of stabilized cultural practice that emerge to mark and perpetuate stabilized perception–action dynamics between people. Unlike motor movements, which are continuously corrected in interaction with the environment (Bernstein 1967), semiotic means – words and symbols – are relatively fixed either as highly repetitive air vibrations or as written inscriptions independent from the context of enactment. As a result, semiotic means can serve as constraints for future dynamics (Rączaszek-Leonardi 2009), allowing an organism to extend situations at hand through time by leveraging non-local historical echoes (Cowley & Nash 2013). Semiotic means expand dynamic coordinations beyond local perception–action practices through communicating with different partners and thus create ecological environments coordinated across different temporal scales (Steffensen & Harvey 2018). Overall, language can "freeze" – in the form of a new semiotic means – the overlapping perceptual orientations toward a shared domain of scrutiny of collaborating individuals in the moments of joint attention. Semiotic activity can then utilize this means to prospectively re-trigger those perceptual orientations even in the absence of temporal synchronization with an interlocutor or even in the absence of any interlocutor, like in reading. Moreover, semiotic stabilizations have consequences for individual cognition: they create a possibility for constraining a person's own dynamics (Abrahamson 2021) and for generalization beyond singular enactment (Bottineau 2010).

« 28 » During semiotic activity, one interlocutor physically alters the environment by sensorimotor production of words and symbols. In that sense, *words act as material tools*, thus unifying traditional distinctions between sensorimotor and semiotic practices. Imagine a friend handing you a spatula while you are cooking together. This motor act might signal to you that it is time to flip over the pancake. As you are familiar with this tool, you immediately appropriate the spatula into your body-artifacts functional system. Similarly, interlocutors "hand" each other words – composed of vibrating air or lines on the screen – to refocus the other's attention and reshape their perception of the environment by highlighting culturally relevant affordances (Goodwin 1994; Bottineau 2010; Van Den Herik 2018). A particular word or metaphor that has stabilized one person's effective perceptual orientation toward a situation can later bring forth target sensations – forward models – for another person operating in a similar situation (Abrahamson 2020). Whereas a precise perceptual re-orientation of one's interlocutor is impossible, interlocutors throw their words into the environment, in an act of *prolepsis*, namely anticipation of the other's ability to bootstrap themselves into a new understanding (Stone & Wertsch 1984). Coupled with the previously stabilized enactment, pre-established semiotic means solicit from the other interlocutor sensorimotor processes *novel for the situation at hand*, and, therefore, potentially usher in contextually effective perceptual orientations.

« 29 » Collaborating peers share similar experiences, and so words can be expected to re-trigger similar-enough target perceptual orientations, thus mediating enactment. In educational situations, however, where, by definition, teachers and learners draw on asymmetric experiences and skills, verbal instruction may fail. Prior to understanding a new mathematical domain, learners lack experience in using domain-specific words and symbols in a culturally expected way. Moreover, they might not be able to establish target perceptual orientations towards shared domains of scrutiny, as their perception is not yet educated (Goodwin 1994; Radford 2010), and may thus fail to distinguish what their teachers say. For example, not only are the students unfamiliar with the "+" sign, they are also unable to identify conjoined segments on a sketch. We thus turn to examine how semiotic mediation operates in educational settings.

Teaching and learning motor skills and mathematics

« 30 » As we noted previously, semiotic means – including formulas and definitions – are cultural artifacts that mark stabilized perception–action dynamics, thus further stabilizing them. Yet, we further proposed, cultural perception–action dynamics related to mathematical concepts are not yet familiar to learners as they come to study the new notions, thus semiotic means *per se* cannot mediate students' understanding. New functional systems of mathematically perceiving and naming the world need to be established.

« 31 » Functional systems are goal-oriented – they are about solving problems. Thus, solving problems lies at the core of any educational process, whether in mathematics or in sports. Sensorimotor problems of sport trainees naturally require developing new sensorimotor processes, which self-organize into stable perception–action loops. While teaching, coaches shape athletes' perceptual orientations through multimodal feedback, commenting on the quality of performance, and feedforward, offering modes of engagement for subsequent trials (Chow et al. 2007; Newell & Ranganathan 2010). Problems in mathematics may seem symbolic or abstract; yet, we suggest, solving a problem comprises senso-motorically coordinating mathematical symbols with properties and structures of the environment, e.g., perceiving graphs of trigonometric functions by attending to the vertical and horizontal positions of the points. As students establish new sensorimotor synergies, new perceptual orientations emerge, which, in turn, enable meaningful (grounded) domain-specific discourse. Just like in sports, mathematics teachers have developed educational strategies to scaffold the students' actions as they develop pedagogically desired sensorimotor synergies and perceptual orientations (see Shvarts & Bakker 2019 for a conceptualization of scaffolding as higher-level regulation in the student–teacher functional system).

« 32 » There are multiple examples in the literature of how teachers can foster target perceptual orientations. Educators might proleptically introduce target discourse (Stone & Wertsch 1984), establishing for the students' sensorimotor system new structural potentialities to be fulfilled through engaging in the learning activities. To educate learners' perception, teachers adjust their own sensorimotor skilled performance to resemble the students' current performance level (e.g., Jermann, Nüssli & Li 2010). Furthermore, teachers exploit multiple modalities for organizing an otherwise highly ambiguous cultural environment, so that students come to distinguish mathematics in it: Teachers use gesture (e.g., Nathan & Alibali 2011; Maffia & Sabena 2020), rhythms (e.g., Radford 2010), and intonations (e.g., Roth 2008). Those direct material interventions *temporarily alter the students' environment* to usher the perception of its problem-relevant mathematical structure. Teachers further support students in reifying their perception of this altered environment through introducing symbolic artifacts. Teachers' sensorimotor communicative means are effective as they directly target students' bodies intercorporeally. In an intercorporeal functional system, collaborators anticipate one another's actions through shared motor intentionality (e.g., the case of football players); it is through such anticipation that students grasp the educators' orienting hints, as they, for example, anticipate the *target* of pointing gestures rather than simply following the gestures themselves (Shvarts & Abrahamson in press). Yet, educators' multimodal expression can be ambiguous, and so learners need to actively search for meaningful coordination between different semiotic means (Shvarts 2018). As a result of teachers' and students' efforts, students finally come to share cultural forms of action and perception for problem-solving in the domain. They further stabilize these forms by way of the semiotic means that they re-establish together with the teachers.

« 33 » A complementary solution for providing mathematics education in line with functional dynamic systems conceptualization comes from the embodied design approach (Abrahamson 2014). In this case, new forms of perception and action are prompted through *the design* of temporarily altered environments – *fields of promoted actions* (Reed & Bril 1996; Abrahamson & Trninic 2015) – even before any mathematical discourse is introduced. In these cases, disciplinary discourse comes *post facto* to reify – mark and further stabilize – already established efficient sensorimotor synergies and mathematically relevant perceptual orientations: In embodied collaboration with teachers, students build semiotic forms on their sensorimotor experiences and emergent perceptual orientations (Flood 2018). Early evidence for the positive impact of this educational approach on classrooms (Alberto et al. 2021; Kosmas & Zaphiris 2023) and remote learners (Shvarts & van Helden 2021) has been encouraging, while pointing to the need for coherent integration into curricula, appropriate professional development, and compatible assessment protocols.

« 34 » Overall, educators prompt goal-oriented activity by organizing learning environments as temporary transformations – by extensive multimodal semiotic expressions or by educational designs – of the culturally normative ecological niche. The creation of such environments has been broadly conceptualized as a type of *prolepsis* (Cole 2016) – a social form of intentionality that teachers and adults exhibit as they pass cultural forms of perception and action over to future generations, thus handing down the culture, along with its material artifacts and their mediating capacity. In a sense, those temporarily altered environments are a society's forward models – a large functional system that tends to reproduce itself unless new global aims are in place calling to revisit national educational agendas (Petitmengin 2021).

Empirical finale

« 35 » Let us consider the mathematical problem of finding a value *a* that satisfies the equation $\sin a = \sin 2a$. Before reading on, please consider for a moment how you would go about solving this problem. There could be multiple approaches. One could use a known formula for the double angle to solve the equation algebraically. Another approach could be to draw two graphs and calculate where they intersect. Yet a third alternative is to imagine two angles, *a* and 2*a*, on a unit circle and figure out at which positions their sine values match. In our study, a tutor scaffolded a student in solving this equation using a unit circle, per this third approach. Dual-eye-tracking technology combined with videography allowed us to investigate intercorporeal coordinations of participants' sensorimotor processes (gazes and gestures).

« 36 » The analysis of empirical data builds on ethnomethodological conversation analysis of multimodal behavior as a tool to understand learning (Abrahamson et al. 2019) extended by the analysis of eye-movement patterns within and across two bodies. As our theoretical stand assumes any behavior is driven by multilevel intentionality, we describe behavior as such, thus relying, on the one hand, on our natural capacity as educators to make sense of what a student and a tutor do, and, on the other hand, interpreting it through the lens of our theory.

« 37 » Working in a technological environment, participants could move a point up and down along the *y*-axis, thus controlling at once two angles that always necessarily have the same sine value (corresponding to the green vertical projection, see Figure 1). Through manual exploration, one can determine a position where the two angle measures relate as 1:2. Analysis of eye-movements revealed specific gaze movements along the visual display performed toward arriving at the solution (see Figure 1).

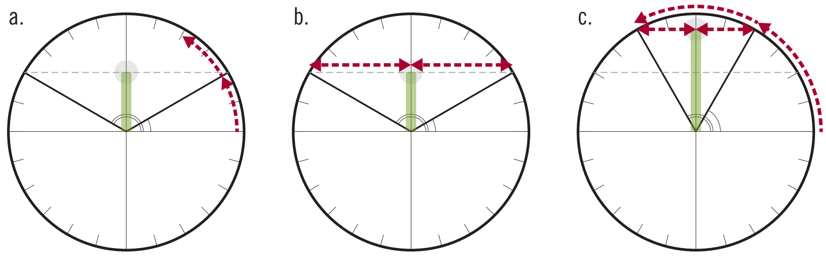


Figure 1 • Based on screenshots from an interactive technological environment for studying trigonometric functions on a unit circle. Red dashed arrows (not visible during teaching/learning) trace gaze movements that support noticing the two target mathematical relations: (a) doubling an angle; (b) attending to the equal sine values of two angles; (c) adjusting the position of the angles so that both mathematical relations are fulfilled at the same time.

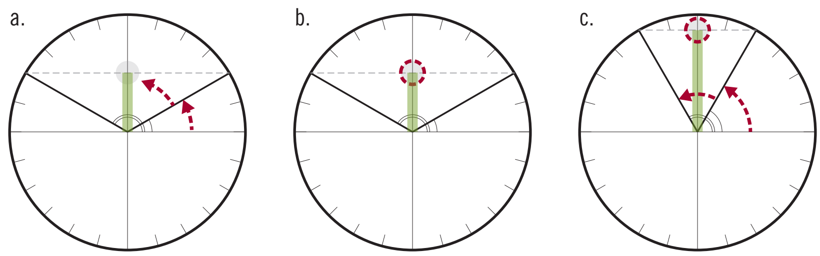


Figure 2 • Alternative way of attending to the same visual display. Red dashed arrows (not visible during teaching/learning) trace gaze movements that support noticing the two target mathematical relations:(a) doubling an angle; (b) attending to the equal sine values of two angels; (c) attending to the adjusted position of the angles without noticing that two mathematical relations are fulfilled at the same time.

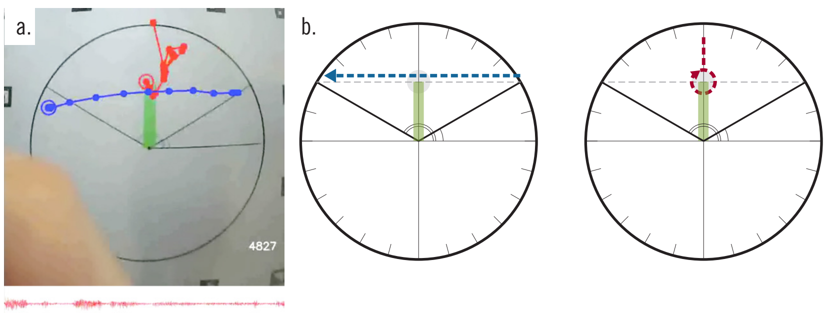


Figure 3 • Different sensorimotor actions on the same visual display while perceiving equal sine values (in all figures, the tutor's gaze is shown in blue and the student's gaze in red): (a) video screenshot with eye-gaze pathway overlays; (b) two schematic reproductions of the video screenshot, showing the tutor and student's respective gaze pathways. Here and further, the direction of the arrows signifies the order of attending to the display.

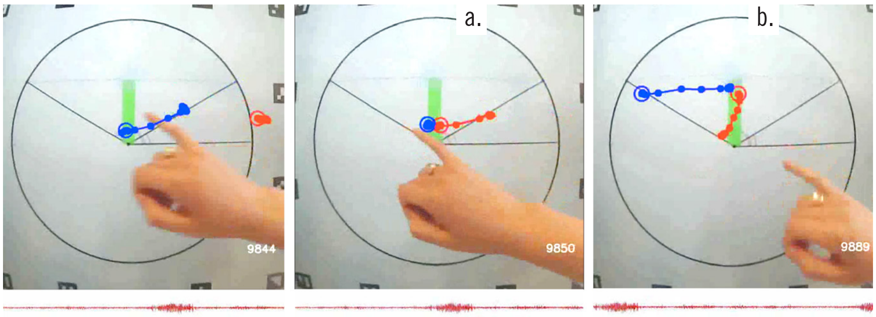


Figure 4 • The tutor gestures to the two angles in question, tracing their radii, while the student follows her attentively. However, their respective eye-gaze movements suggest they are interpreting differently how the diagram expresses the target mathematical relations the tutor is discussing.

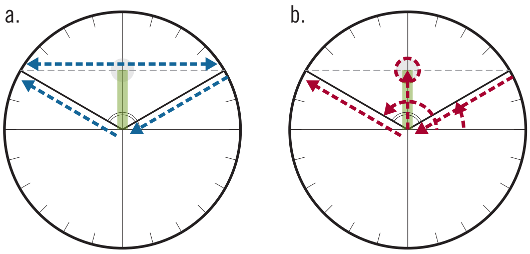


Figure 5 • Discoordination of perceptual structures despite the tutor's verbal–gestural explanation: (a) the tutor attends to two radii as presenting angles and their vertical alignment (b) the student follows the tutor's gesture along the radii yet then reverts to his prior, less-productive perceptual orientation.

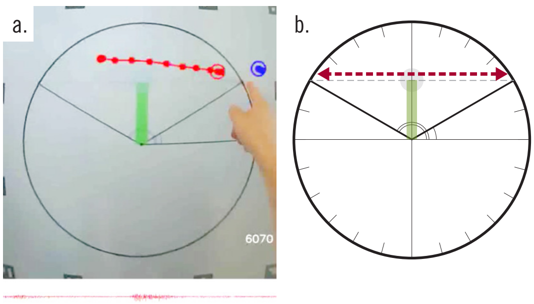


Figure 6 • The student attends to the vertical alignment of the angles for the first time.

« 38 » A pedagogical problem of using a unit circle, however, is that the visual display is ambiguous and the two target relations promoted by the visual display – equivalent sine values, a double angle – can be attended to differently, in a non-functional manner that does not lead to noticing the solution (Figure 2).

« 39 » In our study, the pedagogical problem was resolved through tutorial scaffolding. To begin with, the researcher pointed at the algebraic problem statement $\sin a = \sin 2a$ (a semiotic means placed above the unit circle on the interactive display) to orient the tutor and the student's respective perception of the visual display. However, the eye-tracking data suggest that they were atte C. ing to it differently (Figure 3).

« 40 » The tutor was apparently able to identify the student's idiosyncratic orientation to the display by tightly coordinating her focus with his as she attentively followed his verbal-gestural explanation of the task (as eye-tracking data suggested, not provided here). Next, the tutor, made an attempt to modify the student's attention so that it would match the task-effective perceptual orientation (see Figure 2 in contrast to Figure 1). To do so, she took semiotic measures to explicate her own perceptual orientation to the figure. She gestured along two radii composing an angle and asked whether the two angles in question correspond to the same "height" [sine values]. The student followed her gesture along the two angles (Figure 4a and 4b) but apparently still did not attend to the diagram as she did (Figure 4c). As a result, despite partial coordination with respect to the structures highlighted by their verbal-gestural discourse, the student and tutor perceived the display differently, attending to different diagrammatic features, as eye-tracking data reveals (Figure 5). So, ultimately the student did not foreground the relations that would be productive for solving the problem.

« 41 » Following a few minutes of further struggle, the student begins to explicate why, in his view, the equation cannot possibly be solved. Yet, as he gestures at one angle, he briefly gazes toward the opposite angle (Figure 6), thus for the first time exhibiting the tutor's attentional strategy. This brings him to an insight: "Like the mirrored spot!" Carefully supported by the tutor's synchronized gaze and verbal confirmations in an episode of joint attention (Figure 7), the student practices this attentional strategy a few times and stabilizes it as a new sensorimotor synergy that engages the visual display in a new way. This new sensorimotor synergy enables him to perceive the diagram as coordinated with the algebraic equation, $\sin a = \sin 2a$, simultaneously attending to both the "2" (a double angle) and the "=" (equivalent sine functions). Immediately, he solves the problem.

« 42 » Soon afterwards, the student and the tutor tackle the problem $\sin a = \sin 3a$.

Their gazes are now tightly coordinated when the student refers to the diagram to explain his reading of the algebraic formula. Finding the solution, $\sin 45° = \sin 135°$, the student laughs,[2] immediately joined by the tutor. The participants attended to the multiplied angles and the horizontal alignment of the angles' vertical projections, connecting them by horizontal gaze saccades. This way, the sensorimotor processes, stabilized by the student in a previous task, was effectively adapted to the new task and matches the sensorimotor processes of the tutor. However, while congruent in *space*, the interlocutors' sensorimotor processes were not synchronized in *time*: they exhibit the same spatial articulation of eye-movements yet asynchronously. Nevertheless, the similar perceptual orientations enabled the participants to arrive at the mutual understanding that the target position was found, as their shared laughter evidenced. Now intercorporeally established as a consensual semiotic means, the formula orients the participants on the diagram in a similar manner at different time moments.

« 43 » The mathematical symbols have been mutually established as cueing shared perceptual orientations towards the diagrammatic environment. Within the established intercorporeal functional system, the tutor's semiotic activity influenced the student's sensorimotor processes yet could only partially re-orient the student's perception. It is through his active search for efficient and shared perceptual orientation that the student found an efficient sensorimotor synergy that happened to resemble the teacher's synergy. The interlocutors' shared experience of an efficient intercorporeal coordination becomes "frozen" in the formula, which carries forward this new perceptual orientation through time and contexts.

2 | We acknowledge the great role of the affective component in fulfilling multilevel intentionality through sensorimotor and intercorporeal interactions. However, careful elaboration of the affective dimension goes beyond the scope of this article.

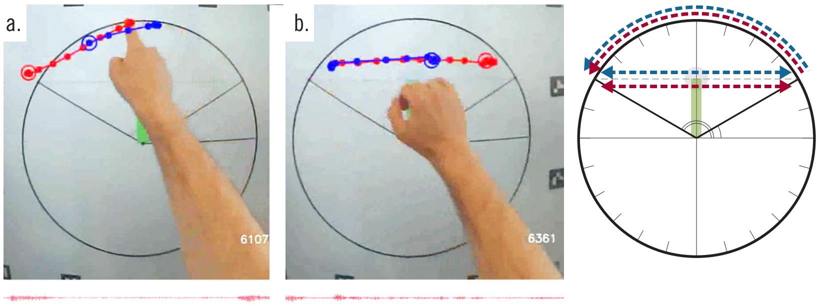
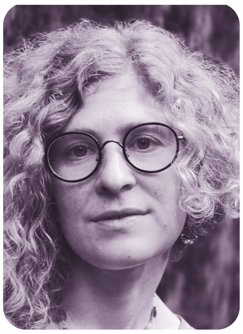


Figure 7 • Spatial–temporal coordination between two bodies, as they come to attend to the visual display in a coordinated manner.

Conclusion

« 44 » In this article we examined semiotic mediation – a higher-level cognitive process, which is often presented in the literature as beyond the reach of a radical embodied explanation in cognitive science. Our theoretical reconsideration of semiotic mediation unfolded over the following tenets:

- We adopt an ecological ontology, in which the presence of a quale is contingent on its relevance for an enactment within an organism–environment system.
- We consider cognitive activity as continuous goal-oriented assembling and excercise of *a functional dynamic system* of sensorimotor processes. The organism's cognitive system develops as the organism strives for a better functional grip on the environment driven by conscious goals and motor intentionality. New skills emerge that solve sensorimotor problems, including sports and mathematical tasks.
- Taken from a functional dynamic perspective, enactment can be understood as a multi-level phenomenon. Emergent stable sensorimotor synergies for organism–environment functional interactions bring forth new contingent forms of perceptual orientation to the environment. Learning achieves a better functional grip, thus coming to discern new environmental structures (affordances) that facilitate pedagogically desired forms of action.
- *Cultural artifacts* – such as spoons, sketches, formulas, words, or digital tools – come to reify effective sensorimotor processes and perceptual orientations. Further, they extend the organism's performance forming *body-artifacts functional systems*.
- Social collaboration is an intrinsically *intercorporeal* process, in which sensorimotor and perception–action processes run across different bodies. Striving for a better functional grip on the environment, collaborators with shared motor intentionality anticipate and influence each other's sensorimotor process. As the collaborators come to stable forms of perception–action practices, they may jointly distinguish new environmental aspects that become sources for semiotic reification.
- *Semiotic means* are a subtype of cultural artifacts that capture efficient intercorporeal coordination and may later be reused, thus bridging different situations over space and time. *Semiotic activity* is treated as *physical temporal transformation of the cultural–material environment for the other*. As such, semiotic activity is a top-down introduction of constraints that impact the other's sensorimotor processes and possibly facilitate target perceptual orientation, thus transforming the ecological environment permanently.
- Mechanisms of teaching and learning are the same for physical and mathematical skills. Educators, including designers and teachers, offer temporar-



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ily transformed environments that solicit particular sensorimotor processes and thus invite pedagogically desired perceptual orientations. These shared perceptual orientations are semiotically captured and inducted into disciplinary discourse.

« 45 » Conceptually, our monistic reconsideration of semiotic mediation bridges the Vygotskian cultural-historical approach and the coordination dynamics research field, grounded in Bernstein's ideas. Our approach thus avoids the ontological gap between motor skills and mathematical or linguistic capabilities. The functional dynamic systems approach – as it is extended towards body-artifacts functional systems and intercorporeal functional systems – strives for a reconsideration of higher-order processes without appealing to the notion of mental representation, which is incompatible with contemporary ideas of cognition as emerging in and serving organism–environment interaction. Thus, we contribute to radical embodied cognitive science a dynamic-systems model of semiotic mediation. By stressing the functional constitution of dynamic systems, i.e., their multilevel intentionality, we highlight a perspective valuable for educational considerations. We also highlight the importance of considering perception–action loops as emergent synergetic units of sensorimotor processes available for phenomenological inspection. Analysis of cognition's goal-directedness and of perceptual structures as emergent is critical for dialogue with students and teachers, the end-recipients of our theorization. As an implication for educational practice, our approach highlights and explains the limited efficiency of teaching semiotic activity. We stress the importance of creating problem-oriented material environments for sensorimotor interaction that facilitate the development of perceptual orientations and students' active semiosis. We see knowledge as inherently constituted within organism–environment interaction, where semiotic means – theorized as equally material as other cultural artifacts – come to take part.

« 46 » As we look to the potential futures of learning across the disciplines, we submit that onto-epistemological proposals – here, a theoretical consideration of semiotic activity as a subset of possible material transformations of environments that are constitutive for conceptual knowing – may usher in greater tolerance for how intersectionally diverse students come to grasp and express new ideas (Abrahamson et al. 2019; Benally et al. 2022; Lambert et al. 2022; Tancredi, Wang et al. 2022; Liu & Takeuchi 2023). In particular, designing educational environments as fields of promoted action could bear axiological import by way of better serving students with a variety of grips on the environment. Mathematical knowledge is not in symbols alone (as Harnad's 1990 "symbol grounding problem" suggests). As Piaget maintained,

> "[T]he formation of logical and mathematical structures in human thinking cannot be explained by language alone, but has its roots in the general coordination of actions." (Piaget 1971: 19)

Yet neither is mathematical knowledge in perception–action loops alone. Rather, knowing mathematics emerges through guided coordination of sensorimotor and semiotic activity (Steffe & Kieren 1994), which are inseparable, from our perspective. As interactive digital resources and classroom epistemic climates come to attune to a functional-dynamic-systems view of mathematics teaching/learning, future education may acknowledge students' idiosyncratic sensorimotor processes and, thus, contribute to an equitable inclusive society.

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Competing interests

The authors declare that they have no competing interests.

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