Dynamic Connections between Problem Posing and Problem-Solving: On the Usefulness of Multiple Perspectives

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> Upshot • Drawing from the ideas of Varela, Proulx and Maheux, I propose a theoretical framework to examine problem-posing and problem-solving and provide evidence for their ideas with examples of student work from their research studies. I will draw comparisons between the approach taken by the researchers to the constructivist approach I have taken in my studies of problem-solving and those conducted with collaborators. My intent with these comments is not to argue the merits of one perspective over the other; rather, I look to point out and elaborate on these differences and make some specific suggestions to the researchers.

« 1 » I enjoyed reading the target article by Jérôme Proulx and Jean-François Maheux: It gave me much to think about as I continue to conduct research in this area. In addition to learning about the ideas of Francisco Varela (and inspiring me with motivation to learn more), I found myself agreeing with the article's rationale that questions the usefulness of the "toolbox" metaphor to characterize problem-solving (§1) as well as one of the main conclusions of the article, that problem-posing and problem-solving can and should be seen as related (§3). I will comment on each of these by making some comparisons to my own work and work with collaborators in this area and, in so doing, will make recommendations to the researchers for their future analyses.

« 2 » Concerning the target article's rationale, I agree with the researchers' assertion that the toolbox metaphor is an inadequate characterization of problem-solving even though it does appear to frame many studies of problem-solving. The idea that a problem-solver merely selects a strategy to apply to find a solution is overly simplistic, as pointed out by the researchers (§1). This point has been made by other researchers, most notably by James Hiebert et al. (1996), who proposed instead a reflective inquiry model that builds on John Dewey's view that "all reflective inquiry starts with a problematic situation" (Dewey 1929: 189).

« 3 » In §§6–9, Proulx and Maheux elaborate on what it means for students to pose and solve a problem. I think the researchers could improve their analysis by explaining, in detail, how the solver's transformation of the prompt (§7) leads to problem-posing and problem-solving (§8) because in addition to providing a more complete explanation of how solvers "*transform* the prompt into a mathematical problem for themselves" (§7), such an analysis will shed light on the informal and subjective actions of the solvers. In particular, it would be useful to know the extent to which the problem task listed in Box 1 (§6) presented genuine challenges to the problem-solvers.

« 4 » The consideration of the solvers' sense of problematics of the situation is important because it helps to distinguish between minor problems that may be routine exercises for the solver and more complex problems that require further definition and exploration by the solver. For example, Gordon Pask (1985: 79) argued that the best problems seldom come "pre-packaged" and already well-formulated, but rather as "problematic situations" that the individual experiences in the course of on-going activity (ibid). Raffaëlla Borasi (1986: 138) also cited the benefits of engaging students in "problematic situations," because they invite additional exploration and problem-posing in the form of problem re-formulation. Jean Lave (1988) makes a similar point in her criticism of the research on algebra word problems, which she believes are more like puzzles than genuine problems, and claims that problems are experienced by solvers as "dilemmas" in the course of on-going activity (Lave 1992: 80). Lave concludes that an important goal for researchers should be to explain the ways that solvers successfully resolve these dilemmas.

« 5 » While the authors' citations in §§2 and 4 suggest the importance of observing the on-going actions of solvers engaged in problematic situations, Leslie Steffe provides a constructivist explanation of what it means to say that a student experiences a problem situation.

> "If a student assimilates a situation using a concept image and, as a result of the assimilation, establishes a goal but can find no procedure in the concept to reach the goal, the student would be in a state of activation with no means to relieve the involved perturbation. There would be an unrelieved tension – a lack of goal satisfaction that some might express as 'it still bugs me!' Polya would say the student has a problem and I concur." (Steffe 1988: 49)

« 6 » Steffe's characterization is useful since it indicates how a solver may experience a problem while in the process of expressing her conceptual knowledge. In this way, problem-solving situations can be viewed as learning opportunities for the solver to extend her conceptual knowledge.

« 7 » Once Proulx and Maheux have a more complete explanation of how the solver's transformation of the prompt leads to problem-posing and -solving (§§6–9), the problem-solving strategies summarized in Box 1 (§6) become more compelling and persuasive as illustrative examples. While some of these strategies seemed familiar to me based on my own experiences of teaching algebra students, I also note that the collection as a whole represents a rich variety of approaches taken by the students coming from the varied meanings they constructed in interpreting the task. Providing a more complete account of the ways that solvers interact with the prompt (§10) and thus interpret tasks seems in agreement with Ernst von Glasersfeld's call for researchers to account for subjectivity in the meaning-making solvers give to the situations they face.

> "From the constructivist point of view, this must be so, not only because there is always more than one solution to a problem, but also because the problem situations themselves, given that they do not exist independently in an objective environment, are seen, articulated, and approached differently by different cognizing subjects." (Glasersfeld 1990: 37)

« 8 » Relatedly, it would be useful to know how many of the strategies reported in Box 1 resulted from the solver engaging in the resolution of a genuine problem as

defined by the researchers cited in §§4 and 5. For example, the *cross-multiplying* strategy (Box 1, bullet #3) is a popular computational strategy taught in most algebra classes for addressing the fraction coefficient of the equation. I expect that for many students, the original equation would be viewed more as a routine exercise to complete than a genuine problem to solve. In contrast, the other strategies listed in Box 1 appear less frequently in students' solutions of first-degree equations than the *cross-multiplying* strategy and do not appear to correspond to any of the traditional ways students are typically taught to solve first-degree equations. For example, the *finding-a-scalar* and *halving* strategies were most inventive and appeared to involve proportional reasoning by the students. I would like to know more about how the students' reasoning with the task led them to initiate these strategies. I look forward to hearing from the researchers about the different ways these students viewed the original task and how they transformed that initial view of the task into goals for action.

« 9 » I think it is important to account for solvers' individual differences, such as those discussed in §§7 and 8, in the problem-solving analyses we conduct. Through their mathematical actions, solvers try to make sense of and comprehend the problem situations they face, with a view towards constructing relevant relationships and patterns (Ernest 1991). From these conceptions, they may construct goals for purposeful action. They may also entertain and explore new questions that go beyond the original prompt or problem, enabling them to view the original problem from a new perspective. In this sense, problem-posing or re-formulation commences whenever the solvers' understandings are challenged by their evolving interpretations.

« 10 » Concerning the findings reported in the target article, they are compatible with findings from my own studies of problem-solving (Cifarelli 1998, 2010, 2015) as well as those conducted with collaborators (Cai & Cifarelli 2005; Cifarelli & Cai 2005; Cifarelli & Sevim 2014, 2015; Sevim & Cifarelli 2013), which found that problem-posing and problem-solving co-evolve as solution activity commences. I would suggest that our findings and those of Proulx and Maheux provide useful illustrations of Stephen Brown and Marion Walter's (1993) ideas about problem-posing that occurs within problem-solving. Specifically, Brown and Walter asserted that solving a problem presents opportunities to the solver for new questions to emerge, that

> "we need not wait until after we have solved a problem to generate new questions; rather, we are logically obligated to generate a new question or pose a new problem in order to solve a problem in the first place." (ibid: 114)

In this way, problem-posing and problem-solving may be viewed as naturally related in the sense that, in order to solve the original problem, the solver generates additional questions or problems that must be addressed (Cifarelli & Sevim 2015). Edward Silver referred to this kind of problem-posing as "problem formulation or re-formulation [that] occurs within the process of problem solving" (Silver 1994: 19).

« 11 » Our studies are informed by a constructivist view of learning and as such, they address the key issues involved in problem-solving and problem-posing using a different set of theoretical assumptions from those followed by Proulx and Maheux. For example, one of the target article's main conclusions, that we, as researchers, need to move away from "*our* problem and toward *their* problem" (§§26–28), is an excellent suggestion that shifts the focus from how experts might formulate and solve problems to how solvers pose and solve their problems. As constructivists, this approach is a critical starting point for almost all of our studies: we seek to engage students in problem situations, looking to adopt their point of view, to see how they "interpret" and make sense of the problem (Glasersfeld 1987: 12) and to explain the problems they see fit to pose and to solve (see Cifarelli & Sevim 2014 for a thorough discussion of this framework).

« 12 » Despite the critical comments made here, I do see the usefulness of considering multiple perspectives to study problem-solving and problem-posing and so I value the contributions made by Proulx & Maheux. If, as Larry Sowder states, "Problem solving is what you do when you do not know what to do!" (Sowder 1985: 141), then no matter the theoretical perspective chosen, our analytical focus should be on the actions of problem-solvers when they are genuinely challenged and cannot see how to achieve their goals and purposes.

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Co-evolution of Problem Posing and Problem-Solving after Finding a Way In

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> Upshot • The significance of Proulx and Maheux's target article lies in their thorough grounding of some of the ideas of the mathematical problem-posing and problem-solving literature in a strong theoretical framework. They direct our attention to two distinct epistemological assumptions that underlie explanations of problem-solving: the so-called "selection-then-execution hypothesis" and Varela's problem-posing perspective. In this commentary, I will offer two ways their line of research could be extended.

« 1 » In their target article, Jérôme Proulx and Jean-François Maheux clearly introduce and explain Francisco Varela's problem-posing perspective, in which problem-posing is defined as an emergent activity of a solver trying and talking