R involved in solving this problem, on the other. In endo-science, any scientific problem becomes related to *D* and *R* simultaneously: $P=\{D, R\}$ whereas problems in exo-science deal with a specific domain of investigation only: $P=\{D\}$.

- In the traditional model of science, predictions assumed the role of one of the primary goals of the scientific endeavor that scientists should achieve in the course of their research (Casti 1977). Due to this specific goal, economic forecasting models considered the variables from economic systems and the policy arena as exogeneous and, therefore, as explanatory model variables. But economic forecasts were confronted with all sorts of observer effects (Landsberger 1958; Rosenthal 1963, 1966; Sackett 1979), including self-fulfilling and self-destroying prophecies (Merton 1948), all of which can result in false predictions if evaluated by the criterion of distance between actual and predicted values. Within the traditional model and methodology of science, these observer effects are treated as anomalies or as special instances of the impossibility of making reliable predictions in societal domains.

« 8 » In the reflexive endo-mode these observer effects become internalized into the variable set of economic models. Suppose traditional annual economic forecasts predict a very serious fall in GDP per capita, but in the past countervailing measures by the government through massive public investments had offset these predictions such that declines in GDP per capita had been observed. In the traditional frameworks of exo-science, such an example would be treated as an instance of the differences between the natural and the social sciences and the deep differences between a forecast in the natural sciences such as a solar eclipse, which can be predicted with precision and with high reliability, and a prediction in the social sciences such as an economic outlook for the next two years, which turns out to be of deplorably low trustworthiness. In an endo-mode, however, the effects of a published prediction on relevant economic actors, including the government, become part of the forecasting model itself. Here, observer effects of all sorts are internalized and included in the prediction of the economic model which still can turn out to be a reliable and robust forecast, when assessed by the criterion of distances between actual and predicted values. This example shows again that science in a reflexive endo-mode requires a more complex configuration than in the traditional exo-science where observer effects can be simply treated as anomalies.

« 9 » To conclude, "Eigenform and reflexivity" opens the door to many other significant changes and re-directions in the interpretation and in the rule formations for scientific practices, but to discuss them would go beyond the scope of this commentary. It is my strong conviction that an enormous workload waits for researchers interested in and sympathetic to Kauffman's reflexive approach to science, to rewrite the traditional general or discipline-specific methodologies of science into their corresponding reflexive formats.

Karl H. Müller is Director of the Steinbeis Transfer Centre New Cybernetics in Vienna, senior researcher at the Faculty of Social Sciences of the University of Ljubljana and member of the International Academy for Systems and Cybernetic Sciences (IASCYS). His main research interests range from the perspectives and horizons of second-order science and problems of complex modelling or complex data-visualization to the frontiers of radical constructivism and of contemporary RISC-societies.

Received: 14 June 2017
Accepted: 18 June 2017

Eigenform, Symmetry, and the First Distinction

Thomas J. McFarlane
Independent scholar, USA
thomasmc/at/stanfordalumni.org

> Upshot • The intimate connection between eigenform and symmetry is illustrated, providing insight into the relevance of eigenform to physical science. Eigenform is also explored in the context of the first distinction.

Eigenform and symmetry

« 1 » To explore the relation between eigenform and symmetry, let us first recall the meaning of *eigenform*. In general, any transformation *T* (other than the identity) necessarily acts so as to change, in some respect, that which it acts upon. For example, a transformation *T* might change an entity *A* to some altered entity *A′*. We write this symbolically as $T(A)=A'$. However, *T* does not necessarily act so as to change every entity it acts upon. In particular, some special entity *E*, might *not* be changed by *T*. In this case, we write $T(E)=E$. In his target article, Louis Kauffman calls such an entity *E* an *eigenform* and describes it as a form that is invariant with respect to a transformation *T* (§§16f). Because the transformation leaves it fixed, the eigenform is also called a *fixed point* of the transformation *T*.

« 2 » Intimately related to the notion of eigenform is the notion of symmetry. "To look for an eigenform," Kauffman writes, "is to look for something that does not change in the presence of change" (§1). This is also how we look for symmetry. Whenever there is an invariant *E* with respect to a transformation *T*, we call *T* a *symmetry* transformation and we say that an entity has *T*-symmetry when it is invariant under the action of *T*. In other words, *an eigenform of T is an entity with T-symmetry*.

« 3 » For example, an isosceles triangle has reflection symmetry because it is invariant to reflection through a vertical axis. The isosceles triangle is thus an eigenform of reflection. We see that eigenform implies symmetry, and vice versa (Figure 1).

« 4 » Because symmetry plays a fundamental role in science, this intimate connection between eigenform and symmetry allows us to elaborate upon the significance of eigenform in science, particularly in physics. Laws of physics are structures that are invariant to transformations of phenomena. For example, Kepler's second law of planetary motion states that, as a planet moves around in its orbit, the line from the planet to the sun sweeps out equal areas in equal times. This law is valid for all planets and all planetary systems. The law is thus an eigenform of any transformation between systems and planets.

« 5 » In addition, Kepler's second law identifies for each planet a specific invariant quantity (area per unit time) that is unchanged despite the changes in the position and velocity of the planet. The dynamical transformations of the system leave this quantity unchanged. It is an eigenform of dynamical transformation, a structure with dynamical symmetry.

« 6 » There are many well-known symmetries in the dynamical laws of physics. Whenever the dynamical equations of physics exhibit a symmetry with respect to translation in time, there is energy conservation, i.e., energy is an eigenform of temporal translation. Whenever dynamical equations of physics exhibit symmetry with respect to translation in space, there is linear momentum conservation, i.e., linear momentum is an eigenform of spatial translation. These are instances of the celebrated theorem by Emmy Noether (1918), which provides a formal correspondence between continuous symmetry transformations of dynamical laws and conserved (invariant) physical quantities. In other words, *every symmetry transformation of physical laws has an associated eigenform corresponding to a conserved physical quantity*. Conserved quantities in our physical theories are eigenforms.

« 7 » In the historical development of physics, we can observe a development of laws applicable to wider empirical domains and a corresponding emergence of deeper symmetries and conserved physical quantities (eigenforms). For example, with the development of special relativity, the symmetries of Galilean transformations were generalized to Lorentz transformations. This group of symmetry transformations was further extended by general relativity. These groups of transformations have their associated eigenforms. The practice of physics may be seen as the quest for deeper eigenforms of rigorous scientific modes of empirical observation, eigenforms that may appear to us as if they correspond to objective physical entities. Deeper symmetries correspond to more abstract eigenforms, further removed from the eigenforms of ordinary experience.

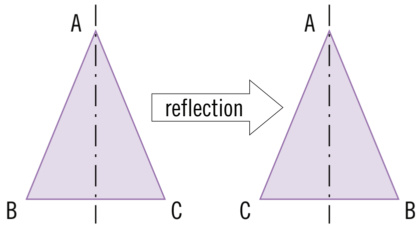

Figure 1 • An isocleles triangle is an eigenform of reflection.

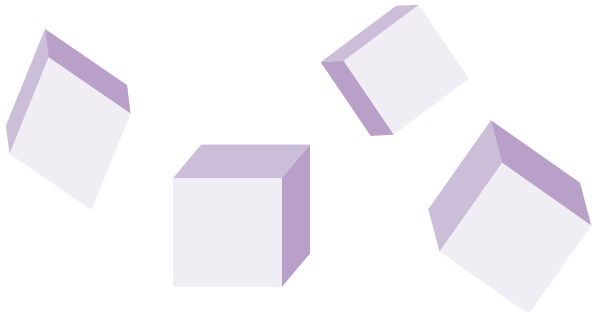

Figure 2 • The 3D structure of box is an eigenform of change in perspective.

« 8 » Albert Einstein (1956: 59) has noted that "The whole of science is nothing more than a refinement of every day thinking." We have seen that physics provides a rigorous mathematically-described illustration of how apparently objective entities correspond to eigenforms of scientific modes of observation. This is a refinement of a similar process whereby objects of ordinary experience emerge as eigenforms of our processes of observation. As Kauffman wrote, "*Ordinary objects are invariances of processes performed in the space of our experience*" (§17).

« 9 » The visual appearance of a box resting on a table, for example, may be viewed as an eigenform of visual perception. But the appearance is not an eigenform if we allow change in perspective (Figure 2). There is a deeper eigenform, however, that corresponds to the three-dimensional structure of the box. The abstracted three-dimensional properties of the box constitute a structure that is an invariant of not just a single observational mode, but a group of observational modes corresponding to different perspectives. Ordinary experience of three-dimensional objects thus emerges as a refinement of visual appearance.

« 10 » Science seeks to refine ordinary experience in several ways. For example, scientific practice requires the use of measurement devices calibrated to standardized units of measurement, and it requires that measurement procedures be capable of reproduction in different times and places. The practice of science, in effect, represents a restriction of all possible observational modes to a group of admissible observational modes. As elaborated by Joe Rosen (2008), these constraints of scientific observation may be viewed as symmetries that define science and the domain of its applicability. These observational modes of science then correspond to particular eigenforms. What is commonly viewed as the objective physical world thus appears as the eigenforms corresponding to the collection of those observational modes that are admissible by the conventions of scientific practice.

Eigenform and the first distinction

« 11 » While scientific practice is an exploration of a restricted class of refined observational modes and their associated eigenforms, we can, alternatively, seek the simplest and most general class of obser-

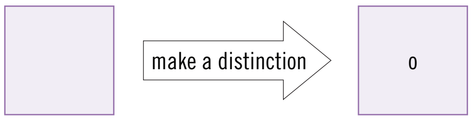

Figure 3 • Marking a page distinguishes marked from unmarked states of the page.

vation. As discussed in McFarlane (2006), many philosophers over the centuries have noted that, in order for anything to be observed or identified or experienced, it must first be distinguished. To indicate something entails distinguishing it from what it is not. Distinction thus appears at the root of all observational acts.

« 12 » Note that, in identifying distinction itself as the root of observation, we have necessarily invoked distinction. Distinction is implicit in the act of distinguishing everything, including distinction itself. The snake bites its tail. Distinction spontaneously self-arises reflexively as both an action and the result of that action. When a space is marked, the marking and the mark are identical. As Kauffman writes, "The sign indicates the very distinction that the sign makes" (§34). Reflexivity is thus at the root of every possible observation, form, or experience.

« 13 » We may ask, what is the eigenform of the first distinction? Viewing distinction as a transformation, we seek that which is invariant to that transformation. The eigenform of distinction is thus that which does not change under the act of distinction. We may view this eigenform as the context or domain *D* of the first distinction. The invariance of *D* under the action of distinction, $\langle\ \rangle$, may be expressed as $\langle D \rangle = D$. Because *D* is unchanged by distinction, it is incapable of even being distinguished from itself. It is, as it were, immune from division or duality. The act of distinction thus, paradoxically, is a non-action, creating no actual change or distinction in *D*. The form of distinction dissolves, just as mysteriously as it had apparently arisen.

« 14 » Everything is based upon distinction, yet distinction has been revealed by its eigenform to be only apparent. Yet, we may nevertheless view distinction *as if* it were actual, as a transformation that creates and exists as a stable distinction between itself and that which it is not. Viewed this way, distinction creates and indicates itself (explicitly) together with its own opposite (implicitly). Form stabilizes as duality between a distinguished state and non-distinguished state.

« 15 » Following George Spencer Brown (1969), the above may be illustrated by making a mark upon a page (Figure 3), which is viewed as distinguishing a marked state of the page (left) from an unmarked state of the page (right). The mark itself makes the distinction.

« 16 » Having constructed stabilized form, the stage is then set for the creative unfolding of endless forms within this context. Insofar as this unfolding is predicated upon the stabilization of the first distinction, we may view the first distinction as being invariant to all derivative acts of distinction and observation. It is a foundational eigenform of subsequent transformations of forms upon the page. The page itself, however, is the deeper invariant. It is the context or domain that is unchanged by even the most elemental and primitive of acts, and is indistinguishable from such acts.

« 17 » Stabilized form and structure emerge by first ignoring the apparent nature of the first distinction and regarding the first distinction as actual. This creates a derivative secondary context for further development of forms of observation and associated eigenforms. By imposing additional specific constraints upon admissible observational modes, particular worlds of eigenform arise corresponding to those constraints. Spencer-Brown's primary arithmetic in Laws of Form is one of the many possible systems of distinction that may emerge. Other examples of such an unfolding, from the first distinction to various mathematical structures, are illustrated in my unpublished manuscript "Distinction and the foundations of arithmetic" (2001), http://www.integralscience.org/lot.html. Such worlds may include mathematical worlds and experiential worlds, which themselves can be refined to evolve more complex worlds. Scientific investigation, and its associated eigenforms, may thus be seen as one possibility for the unfolding of form from the first distinction.

Thomas McFarlane has an interdisciplinary background in physics (BS, Stanford University), mathematics (MS, University of Washington) and philosophy and religion (MA, California Institute of Integral Studies). At the Center for Sacred Sciences in Eugene, Oregon, USA, he lectures and conducts research related to the mathematical and philosophical foundations of science. He is also a patent agent and partner at a patent firm based in Palo Alto, California, USA.

Received: 18 May 2017
Accepted: 8 June 2017

Eigenform and Expertise

Gerard de Zeeuw
University of Amsterdam
dezeeuw.gerard/at/gmail.com

> Upshot • Kauffman proposes to understand scientific thinking as including not only observations but also the act that enables their intentional use. This provides a constructivist opportunity: extending scientific thinking to gaining personal expertise.

« 1 » Louis Kauffman intends to engage "in meta-scientific activity" concerning the question of scientific correspondence (§47): whether what "does not change" in the human environment can be claimed to link or correspond to invariances in human thinking (§1, §16). The answer Kauffman proposes is full of promise. Much work still has to be done, however, to develop its consequences. This suggests an initial effort to look for areas that the proposal may help advance but does not, as yet. These areas will be referred to as "white spots," in memory of the time that Australia and Africa still contained uncharted territory.