Eigenform and Reflexivity

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> Purpose • I introduce the concept of eigenform in the context of second-order cybernetics and discuss eigenform and eigenbehavior in the context of reflexivity. The point of eigenform is that it is a concept arising along with the observer at the point where the observer and the observed are apparently the same and yet apparently different. It is this nexus of the observer and the observed that is central to second-order cybernetics. **> Method** • The article is designed as a formal introduction with excursions into the applications and meanings of these constructions. **> Results** • I show how objects in our immediate experience can be seen to be eigenforms and that in this context such objects are a construct of our interactions, linguistic and otherwise experiential. In this way we can investigate scientifically without the need for an assumption of objectivity. **> Implications** • The implications of this research are important for the performance and exploration of science. We can explore our role in that creation and find that what we create is independent of significant subsets of our actions. The practical implications of this study are strongest for the logical understanding of our constructions and actions. The social implications are in accord with the practical implications. We each produce eigenform models of the others and of ourselves. **> Key words** • **Cybernetics, distinction, circularity, reflexivity, reflexive domain, eigenform.**

Introduction

« 1 » An eigenform is a fixed point for a transformation. In this context an arbitrary transformation is allowed. Transformation means change and when we speak of a transformation, we mean that it is possible for observers to register a change and to begin to describe how that change takes place. For example, we see the transformation of a geological landscape that is wrought by the action of a river that runs through it. We see the canyon and the river far below. We say that course of the river over time has transformed the land and that it has produced the river canyon. The river canyon, while continuing to change, has acquired a form for its observers. This form of the river canyon, seen as a result of the action of the river in the environment is an eigenform. For a certain period of time the river canyon has a recognizable form that is seen to be (in form) not changing under the transformative action of the river. To look for an eigenform is to look for something that does not change in the presence of change. We work with this notion all the time, making objects of the processes in our environments, always changing and making constant objects of the moving display of our perceptions.

« 2 » This idea of fixed point as eigenform is an extension of technical uses of the term, and we mean it to be taken both informally and formally. In the next paragraph, I give a specific example of how an eigenform can arise as the fixed point of a transformation that is simple and syntactical. An eigenform is the analog of an eigenvector in analysis or linear algebra, but it is much more general and includes the fixed points that occur in reflexive domains, as will be explained below.

« 3 » Before using any mathematical formalism, consider the following sentence:

"I am the one who says I."

« 4 » This use of the word "I" is an example of a linguistic eigenform. The word "I" refers to a person, and to the person who is speaking. In this way, I can refer to myself when I say "I say that eigenforms are fixed points." But in the above sentence "I" is itself a fixed point of the phrase

"the one who says."

« 5 » We can rewrite the sentence as

"I am identical with *the one who says* I."

« 6 » And this can become the symbolic

"I = the one who says I."

and in that way "I" is the fixed point or solution to the equation

"X = [the one who says] X."

"$X = F X$."

« 7 » "I" is a fixed point for the transformation F = [the one who says].

« 8 » There are many examples of fixed points in mathematics. For example, the equation $x = 1 + 1/x$ is satisfied by Golden Ratio $\frac{1}{2}(1+\sqrt{5})$.

« 9 » The linguistic eigenform for "I" has many layers of meaning. For example, the eigenform shows how "I" can be taken to refer to any speaker, and thus how there is no preferred "I." Regarding "I" as an eigenform or fixed point is the beginning of a discourse on the meaning of self and self-reference. Other eigenforms can similarly be seen as beginnings, or way stations in a process of understanding. We ask the reader to examine the contents of this essay with these ideas in mind.

« 10 » "I" is an eigenform is of the "interlock" variety. That is, the solution just fits the demand so that the transformation is seen to leave it unchanged. Another example of the same phenomenon is the sentence

"This sentence has X letters."

The sentence becomes true when X = "thirty-three."

« 11 » That is, we have the true sentence

"This sentence has thirty-three letters."

It means exactly what it says.

« 12 » This begins to look like trickery and I agree. But the fact of the matter is that the stability of our perceptions, the arising of objects, concept and symbol is a kind of trickery of the mind. It is a mystery how we go from perception to concept and symbol. In fact, it is an assumption that I choose to discard that there is first signal and then perception, for at the point of perception we also find symbol and concept. In the process of learning (for example, a language) one goes from one form of perception to another, and always one is involved in the actions of engagement that lead to those perceptions. Nevertheless, it may be difficult to appreciate that any object seen in the world is in fact an eigenform resulting from the very quick action of this engagement of perception, symbol, concept. Some aspects of our perception highlight this theme. For example, consider the mirror photograph in Figure 1. The apparently unlimited array of mirrors is seen from the results of light bouncing between two mirrors. Most readers will have experienced this directly and know that the result is essentially instantaneous due to the very high speed of light and the way our nervous system adjusts to the perception. At the cognitive level we idealize the mirrors to an "infinite array" and understand that this array is invariant under the addition of one more level. Here is the appearance in our reality of a mathematical eigenform similar to the nested boxes of Figure 2.

« 13 » In Figure 2 the form of the nested boxes remains the same under the application of one more surrounding box. Putting one more box around the whole figure leaves its form unchanged while adding one more box to the array. *We have gone from counting boxes to seeing the form of the nested boxes.* As long as we do not count, it is as though we have gone to infinity where adding one to infinity does not disturb its infinite nature. We can say that the Nested Squares of Figure 2 satisfies the equation below where we let N denote the Nested Squares.

$N = [N]$ where $[X]$ means "put a box around X."

« 14 » Look at the nested boxes with algebraic symbols. We are playing with letters and parentheses in lieu of making tangible boxes on a table. The letter x stands for anything at all. For example, x could be an elephant and $[x]$ would be an elephant in a box. Algebra takes flight when we release the reference of its "x" to be anything at all in the world of possible things.

« 15 » Let $T(x) = [x]$ define the box transformation. Apply T again and again to an arbitrary x as shown below:

$$x$$
$$[x]$$
$$[[x]]$$
$$[[[x]]]$$
$$[[[[x]]]]$$
$$[[[[[x]]]]]$$
$$\ldots$$

« 16 » If you do this for a long time it begins to look like

$$E = [[[[[[[[[[\ldots]]]]]]]]]]$$

and this expression has the *form of a something that does not change* if you put one more set of brackets around it. Thus E (above) is an eigenform for the transformation T and

$$T(E) = [E] = E.$$

« 17 » The entity E appears via the recursive action of the transformation T. It is a consequence of how one deals with E, seeing it as invariant under T, that makes it into an object for our perception. That E appears as an object is part and parcel of being an eigenform. Thus, Heinz von Foerster spoke of eigenforms in the phrase "objects: tokens for (eigen-)behaviors" (Foerster 2003). Von Foerster, in a wonderful turn of perception, turned the mathematical idea on its head. He pointed out that ordinary objects are tokens for eigenbehaviors. *Ordinary objects are invariances of processes performed in the space of our experience.* The "space of our experience" is the context in which our experience appears. The space of our experience is the experience itself. We make objects by finding fixed points in the recursion of our interactions. Eigenforms are a touchstone for the relationship of circular and recursive processes and the ground of our apparent worlds of perception. This point of view should be compared with that of Humberto Maturana and Francisco Varela (1987), where they see the construction of linguistic domains through the coordination of linguistic coordinations of actions of organisms in the course of their evolution.

Figure 1 • Photograph from facing mirrors.

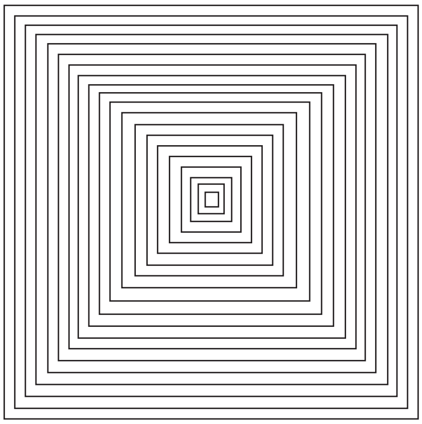

Figure 2 • Nested boxes.

Reflexive domains

« 18 » In previous papers I have discussed the notion of *reflexive domain* as an abstract description of a linguistic domain in which cybernetics can occur. Each participant in the reflexive domain is also an actor who transforms that domain. In full reflexivity, each participant is entirely determined by how he or she acts in the domain, and the domain is entirely determined by its participants.

« 19 » Given an element or person P in a reflexive domain D, that person interacts with other persons Q to form new persons PQ (P acting on Q) or QP (Q acting on P). In this way we can think of each person P in the reflexive domain as a mapping of the domain to itself such that a given Q is transformed into PQ by P.

« 20 » We denote the actions of the domain D on that domain D by $[D, D]$. Then the reflexivity of the domain, that actors and entities are one and the same, is indicated by the identity.

$$D = [D, D].$$

(See Kauffman 1987, 2001, 2004, 2005, 2009, 2012a, 2012b, 2015; Scott 1971; Varela 1979). The symbol $[D, D]$ denotes all the available transformations of the domain D to itself. The equation says that *D is identical to the processes that transforms it to itself.* This is an equation about the entire reflexive domain. If a transformation T is defined by the equation $T(D) = [D, D]$, then we see that a *reflexive domain is itself an eigenform: $D = T(D) = [D,D]$*.

« 21 » A reflexive domain is a context for action, and when we say that the domain D is itself an eigenform, we step back from the domain into a larger context that can include it. This means that no domain, even a reflexive domain, is the end of our deliberations. Each domain can be transcended to a new and larger domain. The process is endless and is the source of all our constructions and considerations.

« 22 » We can draw conclusions for scientific domains from this property, for it is not just an individual result or an object that becomes an eigenform, but rather an entire conversational domain. Focus on some specific language such as English. The domain of English speaking is a reflexive domain and it is capable of reflecting on the language itself. That is, we as speakers of English can discuss English and think and refer to its structure. This in no way perturbs English as a whole. It remains English all the while we the speakers of English, we the representatives of English, speak of English in English. English, as a whole, is not only an eigenform but can be seen to be an eigenform of many of its own transformations!

« 23 » Now consider a pair of actors, Alice and Bob, in a reflexive domain. Let them be denoted by A and by B. Then Alice can imagine Bob and we write this as AB. Bob can imagine Alice and we write this as BA. Each of AB and BA are *themselves* actors in the reflexive domain! There is no loss of individuality in the domain, but each role (such as Bob imagining Alice) also creates a new individual (part of Alice, you might say). Thus, we could have $A' = A(B(A(G(S))))$ which is shorthand for "Alice imagining Bob who imagines Alice imagining George thinking about the stock market." Now A' is herself a new and special individual in the domain.

« 24 » A very important collection of individuals for Alice are the actors $A^*X = A(A(X))$ which are "Alice imagining herself imagining X." These are Alice's states of consciously imagining X. And there is the simple meditative state $A^* = AA$ of "Alice imagining herself." Self-acting states like these are processes. By using reflexive language we have gone to a wider form of speaking about a world and its inhabitants.

« 25 » It will be the subject of further work to experiment with actual use of reflexive language in domains of action. Some of this already occurs in some forms of game theory, but it is possible that we can promote the evolution of "real" reflexive domains by convincing actors to shift their speech. Such speech is well-known to practitioners and students of second-order cybernetics, where at the very least, each person P places herself in the role of the self-conscious P^* in the complexity of the cybernetic social interaction.

« 26 » Now, suppose we define

$$FX = A(XX).$$

FX is an individual who is "Alice imagining (X imagining X)." That is FX is Alice paying attention to the self-reflective process of any individual X. When F pays attention to F, we have Alice paying attention to F paying attention to F, bringing us right back to where we started! Look at the equations.

$$FX = A(XX) \text{ then } FF = A(FF).$$

« 27 » What happened here? F is a form of attention that Alice can bring. And indeed, Alice can bring her attention to how she herself brings attention to her own act of attention. The result is that FF is an eigenform for Alice. Every actor in a reflexive domain is endowed with a self-reflective eigenform that is boot-strapped out of its very own being. We have shown that in a reflexive domain, every entity has a corresponding transformation, and that transformation has a fixed point, an eigenform.

« 28 » We can put this discussion in the form of an "eigenform cartoon," as in Figure 3. This cartoon follows exactly the formalism above with the basic conceit of showing AB with "B" appearing inside the "head" of A. If the reader will take the trouble to visualize the formalism along these lines, then she will see that our reflexivity formalism lines up with much of our common language and visualisation for the division between what goes on "inside" and "outside" of the heads of persons. These are personal fictions that we use to keep the differentiations between one another and the outside world in place. It should now be apparent that once we accept the full use of a reflexive language, then these common distinctions of inside and outside will take on a different flavor. The cartoon itself illustrates this point with its final equation $A(FF) = FF$ where we see that *the entity FF comes into existence in the reflexive space once it is imagined by Alice.*

« 29 » The Eigenform Cartoon illustrates how the fundamental epistemology of individuals is radically different in an articulated reflexive domain. Nothing is more striking than the phenomenon of an individual being identical with the imagination of her by another individual. We can just as easily have two individuals who both have their existence at the imagination of the other. Let $L = BL$ and $B = LB$. Then L depends upon the imagining of B and B depends upon the imagining of L. They are linked eigenforms. If the reader will explore this epistemology, she will find that much of it has appeared in literature of the imagination, and in philosophy. For example, the *L* and *B* linked eigenforms above are an abstract parallel with Chuang Tzu who dreamed that he was a butterfly who dreamed the existence of Chuang Tzu. Indeed, we have from $L = BL$ and $B = LB$ that $L = B(LB)$, that is *L* is identical with *B* who dreams that *L* dreams *B*.

« 30 » It is our hypothesis that we actually live in a reflexive domain with a number of ad hoc restrictions on language (varying from field to field and context to context) that suppress the recognition of the reflexivity. These modes of suppression are used to produce the sorts of results and eigenforms that are desired in given contexts. For example, in mathematics it is taken be the case that any given statement or entity is either true or false. Yet in a very general reflexive domain we can produce an eigenform for negation. Just let Alice be the negation operator (a perfectly good actor in the reflexive world), then $Fx = \sim(xx)$ and $FF = \sim(FF)$. We are no longer in the desired Boolean world. Classical mathematics insists that no entities shall be fixed points of negation. In so doing, classical mathematics shuts the door on the general reflexive domain. This is only part of the story, since one considers non-Boolean domains quite normally in mathematics, but one must mark them carefully to avoid confusion with the classical.

« 31 » The last paragraph touches on a key point in the applications of mathematics to science. Such applications sometimes use radical mathematics involving multiple-valued logic. In such cases one may not be shut out of reflexive domains. But in standard economics, the mathematics is quite classical and there are no fixed points for negation. As a result, the epistemology of standard economic modeling is at the present time primarily shut out of reflexive domains. This is a tragedy, for nowhere is reflexivity more apparent than in the sociological practice of economics, where all actors are affecting one another and the whole.

« 32 » This structural observation has consequences for the applications of cybernetics and applications to the epistemology of second-order science (Kauffman 2015; Müller & Riegler 2014; Umpleby 2014). If science is to be performed in a reflexive domain, one must recognize the actions of the persons in the domain. Persons and their actions are not separate. If an action is a scientific theory about the domain, then this theory becomes a (new) transformation of the domain. *In a reflexive domain, theory inevitably affects the ground that it studies.* The fact that an entire reflexive domain can be seen as an eigenform suggests the observation of that domain in a wider view. For example, physics can be seen as a reflexive domain and one can take a meta-scientific view, allowing physics itself to be one of the objects of a larger domain in which it (physical science) is one of the eigenforms.

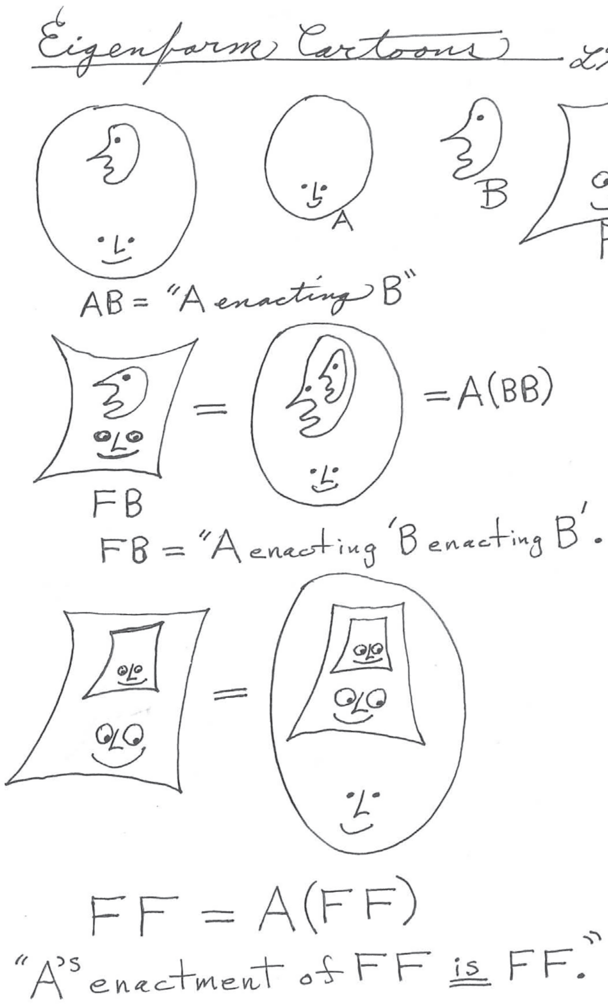

Figure 3 • Eigenform cartoon.

Comments on cybernetic domains and reflexivity

« 33 » Once cybernetics is defined in terms of itself, it becomes what is commonly called "second-order cybernetics." From the point of view of this article, I identify cybernetics as second-order cybernetics and take it (cybernetics) to be itself a reflexive domain. In this way, cybernetics is both the origin of the concept of reflexive domain and is itself an exemplar of that concept.

« 34 » In George Spencer Brown's book *Laws of Form* (Spencer Brown 1969) a very

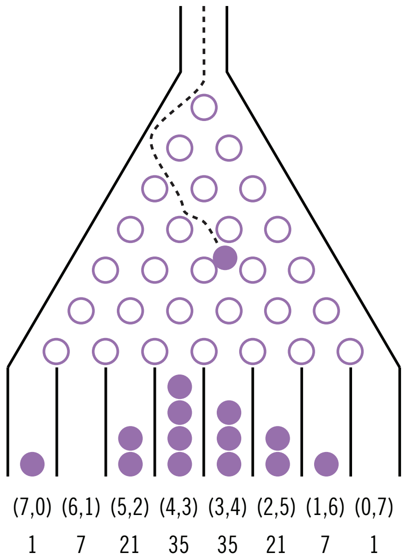

Figure 4 • Coin-tossing probabilities.

simple mathematical system is constructed on the basis of a single sign, the mark, designated by a circle or a box or a right-angle bracket. I shall not develop the formalism here. We can take < > to stand for the Spencer Brown mark. But that very sign, < >, in our eyes, makes a distinction in the plane in which it is drawn. And the interpretation of Spencer Brown's calculus has us understand that *the sign refers to the distinction that the sign makes* (we make that distinction when we are identified with the sign). Thus, the sign of distinction in the calculus of Spencer Brown is self-referential. *The equation*

< > = < >

can be understood as an eigenform equation. We interpret the mark on the left as a transformation from the void of its inside to the marked state that is seen on its outside. We interpret the mark on the right as a mark of distinction. *The identity of the mark of distinction with the act of distinguishing is the fundamental eigenform of Laws of Form.* It is a conceptual fixed point, not a notational one, and this means that there is no excursion to infinity in this eigenform. The sign indicates the very distinction that the sign makes. Everything is said in the context of an observer. The observer herself makes the distinction that is its own sign. In the form, the sign, the first distinction and the observer are identical. The key point about the reflexivity of the sign of distinction in Laws of Form is that it does not lead to an infinite regress. It is through concept that our own thought is kept from spiraling into infinite repetition. Eigenform occurs at the point that infinite repetition is replaced by fixed point and by concept. In this way, our perception is always a precise mixture of sense data and the sense of thought.

« 35 » A deeper relationship with the Fixed Point Theorem (that tells that every *A* in a reflexive domain has an eigenform) is the Russell Paradox. For this purpose, let *AB* denote the relationship "*B* is a member of *A*." Thus, *A* acting on *B* is the act of *B* becoming a member of *A*. Then I can define $Rx = \sim(xx)$ where "~" denotes "not." *R* is a set with the property that *x is a member of R exactly when x is not a member of x. R* is the set of all sets that are not members of themselves. This set *R* is the famous Russell set. The paradox of the Russell set is that RR is an eigenform for negation. For I have

$$Rx = \sim(xx) \text{ and so } RR = \sim(RR).$$

« 36 » This Russellian eigenform is a problem for those who insist that negation must not have fixed points. Certainly, *RR* is a rogue set from the usual point of view since it apparently can both belong to itself and not belong to itself. This depends upon what is the "usual point of view." Looking from the reflexive domain of persons and selves, I see *RR* as a person. *RR* is part of herself, and *RR* is also the whole of herself and so not a part at all. *RR* is a member of *RR* and *RR* is not a member of *RR*. In the reflexive domain of persons and selves it is natural enough for *RR* to be a eigenform of negation. Now we go down the rabbit hole! Let *S* be any set of elements. Define the *Russell Operator R* (Kauffman 2009) by $R(S) = \{x \text{ in } S \mid x \text{ is not a member of itself}\}$. Note that $R(S)$ cannot be a member of itself, since according to its own definition, all members of $R(S)$ are not members of themselves. But therefore $R(S)$ cannot be a member of *S* or it would be a member of $R(S)$ and hence a member of itself. We conclude that $R(S)$ is *not* a member of *S*. No set can be the complete catalog of all sets. And yet we have the concept of "all sets." This concept appears to be an eigenform for the Russell Operator *R*, but we know that "all sets" cannot be a set! Eigenforms may transcend the domains where they have been generated. Eigenforms leave home. But they do not leave our cognitive domains. We have the capacity to handle more than logic would appear to bear.

Eigenform, the shape of probability and the law of large numbers

« 37 » In this section, we examine the law of large numbers relating the standard normal distribution to finite probabilities of equally likely events. The novelty in this example is that one can see from our perspective of reflexivity and eigenform that this law of large numbers expresses an eigenform in the realms of both mathematics and (statistical) experiment. The point of this example is to provide a specific arena to think about how it is that scientific results become repeatable (and even become eigenforms) in the absence of any necessary reality independent of the experimenters. The example also illustrates how theory and experiment act recursively upon each other, and the example indicates how there is much left to be done in understanding how eigenforms arise in the more complex reflexive domains of human social interaction.

« 38 » It is well known that if we repeatedly throw 7 coins and record the number of heads and tails at each toss, then we can calculate the probabilities of the cases $(k, 7-k)$ for each $k = 0, \ldots, 7$ where $(k, 7-k)$ denotes *k*-Heads and $(7-k)$-Tails. These probabilities are illustrated in Figure 4. For example, Probability $(4, 3) = 35/128$. The numbers 1, 7, 21, 35, 35, 21, 7, 1 are the numerators of the probabilities. And in each case the denominator is $128 = 2^7$, the 7th power of the number 2, the total number of outcomes for a toss of seven coins.

« 39 » In Figure 4 is illustrated an experiment with balls dropping through a lattice with equal probability of going left (which we take as corresponding to getting *H*) or right which we take as corresponding to getting *T*). The slots at the bottom correspond to the possibilities of getting *k* Heads and $(7–k)$ Tails. In the experiment, this is a path with *k* Left Turns and (7–*k*) Right Turns. If we use longer paths or more coins, then the numbers change by a simple recursive pattern known as Pascal's Triangle:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

« 40 » Each row is obtained from the previous row by starting with *1* and then successively adding pairs of numbers of the previous row. Thus

$$7=1+6,\ 21=6+15,\ 35=15+20,\ 35=20+15,\ 21=15+6,\ 7=6+1 \text{ and } 1=1+0.$$

We have the *Pascal Recursion*

$$\mathrm{Row}(n+1)=\mathrm{Pascal}\,[\mathrm{Row}(n)]$$

where Pascal[Row(n)] is the result of applying the rule we have just described at an arbitrary row. There is an eigenform for this transformation. It is called the *Standard Normal Distribution* and is described by

$$Y(t) = \frac{e^{-x^2/2}}{\sqrt{2\neq}}$$

The plot of the numbers in the rows of the Pascal Triangle gets closer and closer to this smooth curve *with proper scaling*. See Figure 5 for a depiction of the 40th row of the Pascal Triangle.

« 41 » The result is that we can say that *the Eigenform of the Pascal Recursion is the Standard Normal Distribution.* Here we see how a mathematical eigenform becomes the experimental attractor of many statistical situations whose underlying probability is the analog of equi-probable paths or coin-tossing. It is these eigenform regularities that lie at the base of statistics.

« 42 » We now see both sides of the issue. Eigenforms are part and parcel of the operation of reflexive domains and they occur easily in mathematical and linguistic contexts. Eigenforms, regularities, also occur in our experiments and in our statistics. In the deepest and clearest cases of such correspondences, there is a strong mathematical eigenform behind the scenes, corresponding to the regularity.

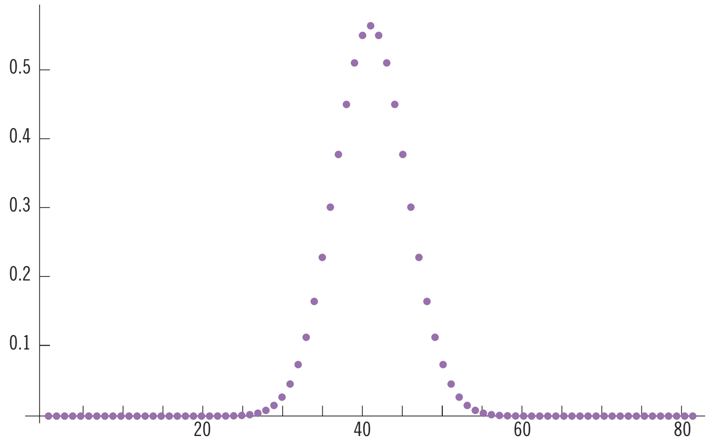

Figure 5 • Row 40 of the Pascal triangle.

« 43 » Note once again how the mathematical eigenform of the Pascal recursion is related to the regularity of statistical observation. In actual practice we use a model at finite accuracy, for example at the level of accuracy of Row 40 of the Pascal Triangle. Actual events will pile up over time and fit the normal curve (the limit eigenform) that is approximated by Row 40. Random events fall, in the realm of large numbers of events, into the pattern of the eigenform. The eigenform becomes the perceived pattern of events, and with the understanding of the mathematics of the central limit theorem, this eigenform becomes conceptual and it becomes a conceptual tool for exploring a world of apparent data. In this we, as observers, stand between our ability to look outward at a world and make records of its events and our ability to correlate those events with numerical and conceptual constructions such as the normal distribution. By finding matches between these internal and external modes of perception, we have built our descriptions of an apparently objective external world, supported by our ability to reason.

« 44 » Can we believe that this will always be the case? Will we be able to rely on mathematical eigenforms to explain the world? We do not know how to do this when reflexivity is strong. We were close to this sort of answer when we looked at the pattern of the Pascal recursion and its approach to continuity. There we had a reflexive model and an understanding of its behaviour in large numbers. Now the task is to go to more robust models of reflexive domains and to begin to uncover their regularities. The reflexive domains contain actors who are the very theorists who create the perceptions that are taken to be the domain that is observed. Chuang Tzu dreams that he is a butterfly who dreams that he is Chuang Tzu. Second-order science will emerge from these considerations as we allow our understanding to unfold into reflexivity.

Conclusion

« 45 » All attempts to find stable knowledge of the world are attempts to find theories accompanied by eigenforms in the actual reflexivity of the world into which one is thrown. The world itself is affected by the actions of its participants at all levels. One finds out about the nature of the world by acting upon it. The distinctions one makes change and create the world. The world makes those possibilities for distinctions available in terms of our actions. Given this point of view, one can ask, as one should of a theory, whether there is empirical evidence for the idea that stable knowledge is equivalent to the production of eigenforms. We have only to look at what we do to see that whenever "something is the case" then there is an orchestration of actions that makes an invariance, making an eigenform for those actions.

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is Professor of Mathematics at the University of Illinois in Chicago. He has a PhD in Mathematics from Princeton University (1972) and a BS in Mathematics from MIT (1966). He has taught at the University of Illinois since 1971 and has been a visiting professor and researcher at numerous universities and institutes. Kauffman is the founding editor and editor in chief of the *Journal of Knot Theory and Its Ramifications* and the editor of the World Scientific Book Series on Knots and Everything. Kauffman is the author of numerous books and research papers on form, knots, and related disciplines. He is a past President of the American Society for Cybernetics, author of the column Virtual Logic for the journal *Cybernetics and Human Knowing*, and a 1993 recipient of the Warren McCulloch award of the American Society for Cybernetics.

« 46 » We have travelled from Ludwig Wittgenstein's dictum that "The world is everything that is the case" (Wittgenstein 1922: §1) to "The world is everything that is eigenformed." And this was already said in Spencer Brown (1969: 1) in the sentence "We take therefore the form of distinction for the form." as we take eigenform to be the form of distinction.

« 47 » Examine scientific endeavors. See how they are interrelated. Find connections among them. Engage in meta-scientific activity. Theories, seemingly objective, affect the world through their very being, and these actions on the world come to affect the theories themselves. In exploring the world, find regularities. These regularities are our own footprint. In the end we shall begin to understand the mystery of the eigenforms that we have created, constructed and found, and understand that the world and ourselves arise together in their creation.

« 48 » In tracing the path of this article, the reader will note that I have passed through many variations on the concepts of eigenform and reflexivity and have concentrated on examples that illustrate these ideas. These ideas have been discussed before and they will be discussed again. The subtlety that lies behind them is only being teased slightly apart by the articulation of the notion of fixed points, eigenforms and reflexive domains. A most important aspect in further creating this form of creating is to understand that expansion of reference leads to contraction of awareness and contraction of reference leads to expansion of awareness. Consequently, we search for the simplest languages (mathematical and linguistic) that can express the exquisite subtlety of the mutual creation of subject and object. At the same time, I am quite willing to examine complex technical or mathematical structures to see how they can participate with us in ultimate simplicity. I am searching for science (knowledge) that is convincing. Distinctions in which we live, such as world and mind, can be examined until their relationships come forth. New formalisms will arise and new worlds will come into being.

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Open Peer Commentaries

on Louis Kauffman's "Cybernetics, Reflexivity and Second-Order Science"

The Tricky Transition from Discrete to Continuous

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> Upshot • I show that the author underestimates the tricky matter of how to make a transition from the discrete, countable to the continuous, uncountable case.

« 1 » Let me first and foremost emphasize that, together with the author, I am deeply convinced of the importance of fixed points, eigenforms and reflexive domains to enhance our understanding not merely of our environment but the environment *whereof we are a part*. Unavoidably this means self-reflection and the "danger" of inconsistencies, antinomies and paradoxes is always present. If indeed these are unavoidable, it is obvious that it is a far better thing to accept this situation as a given and, moreover, to consider it a subject of study. The comments that follow are therefore not meant to reject the author's approach but rather to issue a warning: do not be too generous in the application of your ideas as it unnecessarily weakens the whole enterprise.

« 2 » My focus will be on §§37–44, under the heading "Eigenform, the shape of probability and the law of large numbers." A transformation, labelled *Pascal* is introduced that creates a new row, labelled *Row* in Pascal's triangle, thus obtaining the formula $Row(n+1) = Pascal[Row(n)]$. It is then claimed in §40 that the Standard Normal Distribution (*SND*) is an eigenform for *Pascal*. This I believe not to be the case, as the following paragraphs will try to show.

« 3 » Let me phrase my worries in a more general framework. Pascal's triangle is a special case of the situation where a row of elements is given $e(1), e(2), \ldots, e(n), \ldots$ and a transformation T that allows us to calculate or determine element $e(n+1)$ from $e(n)$, thus $T(e(n)) = e(n+1)$. Two possibilities present themselves. The first one is that, for some k, $e(k) = e(k+1)$. When that is the case. obviously $e(k)$ is a fixed point of T as $T(e(k)) = e(k+1) = e(k)$. It is also easy to show that for all j, $j > k$, $e(j) = e(k)$. This means that, after a finite number of steps, a fixed point is reached and one remains there. The second one states that for all k, $e(k) \neq e(k+1)$. In that case no fixed point can occur but the *Pascal* operation on the rows of the triangle precisely satisfies this condition, meaning that there is no fixed point.

« 4 » There is however a way out of the above situation and in the article, e.g., in §43, the author introduces the concept of "limit eigenform." This means that one adds a special element to the row of elements, say $e(\omega)$, and then stipulates that $e(\omega) = \omega$. Of course, if this were to be a "mere" stipulation, the danger of arbitrariness is present. Somehow one would like to connect the special element $e(\omega)$ to the other elements $e(n)$. And that is, of course, possible. It is precisely the idea of a limit that can provide such a connection. Let me first present an example how this could work.

« 5 » Consider the row of elements $e(n)$ where $e(n) = 1/n$. Define a transformation Fr, such that $Fr(e(n)) = e(n+1)$. In short, we generate the row $1, 1/2, 1/3, \ldots, 1/n, \ldots$ Obviously for any n, $e(n) \neq e(n+1)$. Now add a special element $e(\omega)$, such that $e(\omega) = \lim_{n\to\infty} \frac{1}{n} = 0$. $Fr(e(\omega)) = Fr(\lim_{n\to\infty} \frac{1}{n}) = Fr(0)$. Now, however, we face a difficulty. What is $Fr(0)$ equal to? What is required is some additional condition(s) that can help us move forward. One such condition could be the commutativity of the Fr-operation and the limit operation. Because then we can reason as follows:

$$Fr(0) = Fr(\lim_{n\to\infty} \tfrac{1}{n}) = \lim_{n\to\infty} Fr(\tfrac{1}{n}) = \lim_{n\to\infty} \tfrac{1}{n+1} = 0$$

and hence $Fr(0) = 0$. Of course, in some cases other conditions are possible, the limit operation is just one among many. With this example in mind, let me return to *Pascal*.

« 6 » My claim is that for the *Pascal* example no such condition can exist unless it is stipulated in an arbitrary fashion (and that surely must run counter to the author's intentions). Let us start from the observation that, if the *SND* is a fixed point of *Pascal*, then $Pascal(SND) = SND$. First observe that the transformation loses its original meaning: whatever *Pascal* does to *SND*, it is most certainly not adding another row for that does not make any sense. To what should the row be added and what could it possibly look like? So, there are two processes at work here: not only is *SND* added as an extra element but the transformation itself is also modified. In that case, the question becomes what that modification might be? Note that the limit operation does not help us out here. Suppose that we define *SND* as $\lim_{n\to\infty} Row(n)$. Then we have to determine what the limit can mean here. Since a row in Pascal's triangle has the form $(1, n, n(n-1)/2, \ldots, n!/k!(n-k)!, \ldots, n(n-1)/2, n, 1)$, it seems plausible to take the limit of every element in the row, but apart from the first and last element this will only generate

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infinities. So, the limit operation fails. And I do not see any other condition that could do the job. In short, it is not clear at all how to make a transition from the rows to the *SND* in such a way that it can be proved that the *SND* is a fixed point. Phrased differently, the example of the *SND* as a fixed point does not work.

« 7 » In §1 of this commentary I referred to the author's generosity. He runs the risk of introducing eigenforms as fixed points of transformations everywhere, for if one is too generous, then given a domain D and a transformation T, we can always extend the domain with an additional element d^* and simply stipulate that $T(d^*) = d^*$. But then nothing is gained and one runs the risk of seeing one's enterprise trivialized, which would be a pity as eigenforms do inform us as to how we construct the world.

« 8 » Let there be no misunderstanding: I do know that the binomial distribution is indeed linked to the *SND*. But the derivation of the latter, starting with the former, does not involve fixed points but rather approximations such as Stirling's formula. Any handbook on probability and statistics contains such a derivation. It is inviting to ask the question why that derivation goes through and the "*Row*-and-*Pascal*" machinery does not work. The answer lies, I believe, in the contrast between discrete and continuous. All rows in Pascal's triangle contain a finite number of members, hence the row is countable and forms a discrete set. But the SND is defined over the domain of the reals and that is an uncountable, continuous set. In the classical derivation, one starts with the continuous background straight away and so no transition is needed from the countable (or discrete) to the uncountable (or continuous). After all, it is a huge step to take and it should not surprise us that the transition fails. In a nutshell, when we think of natural numbers, a countable set, it makes sense to ask what is the next number after n, but when we think of real numbers, an uncountable set, the question loses its meaning as there is no next element after a real number r.

« 9 » To make clear how important this distinction is, I would like to mention this fine example. The differential equation $df/dt = f \cdot (N - f)$, where f is a function of t and N a given constant, in the continuous case has the logistic curve as a solution, leading to an equilibrium point N, which could be interpreted as an eigenform of that curve. (Technically one can say that there is a moment t^* such that for $t > t^*$ and $t' > t^*$, $f(t) \approx f(t')$, where "≈" expresses near-equality.) However the discrete version of the same equation, namely $\Delta f / \Delta t = f \cdot (N - f)$, where t (and hence f) now take discrete values, shows chaotic behaviour. It is not clear at all where an eigenform can be found here. More details can be found in Van Bendegem (2000).

« 10 » To conclude, I believe that the distinctions between the countable and the uncountable, between the discrete and the continuous should be the next topics on the research agenda for the study of fixed points of transformations or eigenforms. It would probably allow us to get a clearer (and perhaps richer) picture of how to relate the (countable) Pascal's triangle to the (uncountable) *SND*.

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Fueling the Take-off Stage of Scientific Reflexivity

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> Upshot • I emphasize the significant role played and the pioneering efforts that Kauffman has made over the last thirty years to present a full-fledged new and reflexive model of science that offers itself as a path-breaking alternative to the traditional models and methodologies of classical philosophy of science.

« 1 » In his well-known model, Walt Rostow (1971, 1978) distinguishes five stages of socio-economic modernization: traditional society, preconditions for take-off, take-off, drive to maturity, and age of high mass consumption. The take-off phase marks a spectacular period of growth and the successful transition from traditional societies to modern ensembles. During the take-off stage, the components of the traditional society are vanishing rapidly and the elements of modern societies are quickly emerging.

« 2 » Likewise, Louis Kauffman's target article "Eigenforms and Reflexivity" can be viewed as a highly significant component for the take-off stage in the rise of scientific reflexivity that characterizes the transition from the traditional conceptions of science to a new and reflexive structuration and organization of scientific processes. The traditional model of science can be characterized as the "London consensus," following the International Colloquium in the Philosophy of Science in London in 1965 (Lakatos 1967, 1968; Lakatos & Musgrave 1968, 1970). It can be seen as the culmination and, at the same time, as the turning point of traditional philosophy of science. Philosophers of science such as Rudolf Carnap (1966) and Karl Popper (1963) can be viewed as the epitome of this traditional model of science, with its emphasis on objectivity, rigorous tests and theory-driven dynamics. The presence of Thomas Kuhn at the London Colloquium and his insistence on the relevance of the history and sociology of science pointed already to substantial revisions of the traditional model and the methodology of science. They were further enhanced and widened by sociologists of science with a new wave of laboratory studies. As the following anecdote suggests, Kauffman was already well embedded during the period of the demise of philosophy of science models and in the formation of alternatives to the classical methodology of science.

« 3 » On 5 February 1993, Heinz von Foerster (1911–2002), one of the pioneers for a non-traditional perspective of science and its general methodology (Müller & Müller 2007), gave a lecture in honor of Niklas Luhmann (1927–1998). Von Foerster brought three presents for Luhmann which he considered a "trio of American jewels" (Foerster 2003: 307).

- The first jewel was a short article by Warren McCulloch (1898–1969), "A Heterarchy of Values Determined by the Topology of Nervous Nets." In this article Warren McCullough dealt with a significant feature of our cognitive organization, namely the circularity of preferences which led him to an astonishing consequence: "For values, there can be no common scale" (McCulloch 1988: 43).
- The second jewel was a short article by Karl Menger (1902–1985), a mathematician with strong ties to the Vienna Circle. In "Gulliver in the Land without One, Two, Three" (Menger 1979), Menger developed the idea of "assimilating the symbols for the simpler functions to those of more complicated ones instead of the other way round" (ibid: 312), just as Gulliver proposed in the island, with a stick-notation for 1, 2 and 3 and numerals elsewhere to obtain uniformity in the numerical system "by assimilating their lower to their higher numerals" (ibid: 307).
- Finally, the third jewel was Kauffman's "Self-reference and recursive forms" (Kauffman 1987), who described his article as a "panoply of fundamental mathematical and physical ideas relating directly to the central turn of self-reference" (ibid: 53).

For von Foerster, all three jewels were significant components for second-order cybernetics, which he envisioned as a much broader dialogic model for science that includes the participant scientists as necessary actors and proposed eigenforms as the goal of scientific explorations.

« 4 » 30 years later we are offered another jewel from Kauffman. It expands and widens his earlier work and can be considered a sophisticated and fully developed version of von Foerster's vision of second-order cybernetics as a new and alternative type of inclusive or reflexive science. It is an important article for everyone who takes science, scientific work and scientific methodology seriously.

Traditional science models and their general methodologies (exo-science)	Reflexive science models and their general methodologies (endo-science)
Researchers implicit (excluded)	Researchers explicit (included)
Elimination of subjective goals	Specification of expectations, preferences, etc.
Value-free	Explication of values by researchers
Objectivity	Eigenforms
Theory-driven	Various drives, including theoretical ones
Tests and experiments	Tests and experiments as special eigenforms*
Results of tests and experiments as the basis for the rejection or provisional acceptance of hypotheses, theories, etc.	Results of tests and experiments as the end-points of recursively closed dialogues between researchers and, if feasible, the domain of investigation
Explanation, prediction, control	Eigenforms of various types*
Observer effects as anomaly	Observer effects as being included
Classical philosophers of science including Carnap and Popper as leading proponents	Von Foerster, Humberto Maturana, and Kauffman as seminal figures of the reflexive approach

Table 1 • Basic distinctions between traditional and reflexive models of science and their general methodologies. (*) Following Kauffman eigenforms can be reached in numerous forms or types. A group of psychologists could discuss, for example, the design of a psychological test and could reach full consensus on the specification of this test which would then result in a particular test as eigenform under an intrinsic composition of researchers. A group of physicists could agree on the design of an experiment and this consensus would lead to an agreed-upon experimental design under a specific personal composition as an eigenform. In this sense eigenforms can be conceived in numerous types, like experiments, tests, measurements, explanations, descriptions, etc.

« 5 » "Eigenform and reflexivity" provides a full outline for a radically different model of scientific operations and for a new general methodology of science that is no longer organized as an interplay between "conjectures and refutations" (Popper) or as a search for "objective knowledge," but as a consensus driven enterprise composed of inclusive researchers operating in reflexive domains and producing eigenforms as the results of these recursive interactions. The traditional methodology of science, as developed by Carnap (1966), Carl Gustav Hempel (1966), Ernest Nagel (1961), and Popper (1975), should be seen as the target domain to be fully replaced by Kauffman's alternative description of the organization and structures of scientific processes.

« 6 » The importance and the relevance of "Eigenforms and reflexivity" can be made more apparent by suggesting a general dichotomy that indicates clearly a break or a distinction between traditional ways of doing and practicing science and its general methodology and the new reflexive and inclusive forms that Kauffman suggests. Table 1 presents the necessary conceptual distinctions between the traditional view of science and its methodology under the label of exo-science or "science from without" and introduces the new term of endo-science or "science from within" as a useful way of describing the new reflexive approach promoted by Kauffman.

« 7 » I would like to stress two points that show the incisive differences between the traditional methodology of science or, alternatively, of exo-science and its reflexive counterpart or endo-science.

- Endo-science is more complex and more challenging than traditional exo-science because in endo-science each scientific problem *P* is composed of two areas, namely the domain under investigation *D*, on the one hand, and the researchers

R involved in solving this problem, on the other. In endo-science, any scientific problem becomes related to *D* and *R* simultaneously: $P = \{D, R\}$ whereas problems in exo-science deal with a specific domain of investigation only: $P = \{D\}$.

- In the traditional model of science, predictions assumed the role of one of the primary goals of the scientific endeavor that scientists should achieve in the course of their research (Casti 1977). Due to this specific goal, economic forecasting models considered the variables from economic systems and the policy arena as exogeneous and, therefore, as explanatory model variables. But economic forecasts were confronted with all sorts of observer effects (Landsberger 1958; Rosenthal 1963, 1966; Sackett 1979), including self-fulfilling and self-destroying prophecies (Merton 1948), all of which can result in false predictions if evaluated by the criterion of distance between actual and predicted values. Within the traditional model and methodology of science, these observer effects are treated as anomalies or as special instances of the impossibility of making reliable predictions in societal domains.

« 8 » In the reflexive endo-mode these observer effects become internalized into the variable set of economic models. Suppose traditional annual economic forecasts predict a very serious fall in GDP per capita, but in the past countervailing measures by the government through massive public investments had offset these predictions such that declines in GDP per capita had been observed. In the traditional frameworks of exo-science, such an example would be treated as an instance of the differences between the natural and the social sciences and the deep differences between a forecast in the natural sciences such as a solar eclipse, which can be predicted with precision and with high reliability, and a prediction in the social sciences such as an economic outlook for the next two years, which turns out to be of deplorably low trustworthiness. In an endo-mode, however, the effects of a published prediction on relevant economic actors, including the government, become part of the forecasting model itself. Here, observer effects of all sorts are internalized and included in the prediction of the economic model which still can turn out to be a reliable and robust forecast, when assessed by the criterion of distances between actual and predicted values. This example shows again that science in a reflexive endo-mode requires a more complex configuration than in the traditional exo-science where observer effects can be simply treated as anomalies.

« 9 » To conclude, "Eigenform and reflexivity" opens the door to many other significant changes and re-directions in the interpretation and in the rule formations for scientific practices, but to discuss them would go beyond the scope of this commentary. It is my strong conviction that an enormous workload waits for researchers interested in and sympathetic to Kauffman's reflexive approach to science, to rewrite the traditional general or discipline-specific methodologies of science into their corresponding reflexive formats.

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Eigenform, Symmetry, and the First Distinction

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> Upshot • The intimate connection between eigenform and symmetry is illustrated, providing insight into the relevance of eigenform to physical science. Eigenform is also explored in the context of the first distinction.

Eigenform and symmetry

« 1 » To explore the relation between eigenform and symmetry, let us first recall the meaning of *eigenform*. In general, any transformation *T* (other than the identity) necessarily acts so as to change, in some respect, that which it acts upon. For example, a transformation *T* might change an entity *A* to some altered entity *A´*. We write this symbolically as $T(A) = A'$. However, *T* does not necessarily act so as to change every entity it acts upon. In particular, some special entity *E*, might *not* be changed by *T*. In this case, we write $T(E) = E$. In his target article, Louis Kauffman calls such an entity *E* an *eigenform* and describes it as a form that is invariant with respect to a transformation *T* (§§16f). Because the transformation leaves it fixed, the eigenform is also called a *fixed point* of the transformation *T*.

« 2 » Intimately related to the notion of eigenform is the notion of symmetry. "To look for an eigenform," Kauffman writes, "is to look for something that does not change in the presence of change" (§1). This is also how we look for symmetry. Whenever there is an invariant *E* with respect to a transformation *T*, we call *T* a *symmetry* transformation and we say that an entity has *T*-symmetry when it is invariant under the action of *T*. In other words, *an eigenform of T is an entity with T-symmetry*.

« 3 » For example, an isosceles triangle has reflection symmetry because it is invariant to reflection through a vertical axis. The isosceles triangle is thus an eigenform of reflection. We see that eigenform implies symmetry, and vice versa (Figure 1).

« 4 » Because symmetry plays a fundamental role in science, this intimate connection between eigenform and symmetry allows us to elaborate upon the significance of eigenform in science, particularly in physics. Laws of physics are structures that are invariant to transformations of phenomena. For example, Kepler's second law of planetary motion states that, as a planet moves around in its orbit, the line from the planet to the sun sweeps out equal areas in equal times. This law is valid for all planets and all planetary systems. The law is thus an eigenform of any transformation between systems and planets.

« 5 » In addition, Kepler's second law identifies for each planet a specific invariant quantity (area per unit time) that is unchanged despite the changes in the position and velocity of the planet. The dynamical transformations of the system leave this quantity unchanged. It is an eigenform of dynamical transformation, a structure with dynamical symmetry.

« 6 » There are many well-known symmetries in the dynamical laws of physics. Whenever the dynamical equations of physics exhibit a symmetry with respect to translation in time, there is energy conservation, i.e., energy is an eigenform of temporal translation. Whenever dynamical equations of physics exhibit symmetry with respect to translation in space, there is linear momentum conservation, i.e., linear momentum is an eigenform of spatial translation. These are instances of the celebrated theorem by Emmy Noether (1918), which provides a formal correspondence between continuous symmetry transformations of dynamical laws and conserved (invariant) physical quantities. In other words, *every symmetry transformation of physical laws has an associated eigenform corresponding to a conserved physical quantity*. Conserved quantities in our physical theories are eigenforms.

« 7 » In the historical development of physics, we can observe a development of laws applicable to wider empirical domains and a corresponding emergence of deeper symmetries and conserved physical quantities (eigenforms). For example, with the development of special relativity, the symmetries of Galilean transformations were generalized to Lorentz transformations. This group of symmetry transformations was further extended by general relativity. These groups of transformations have their associated eigenforms. The practice of physics may be seen as the quest for deeper eigenforms of rigorous scientific modes of empirical observation, eigenforms that may appear to us as if they correspond to objective physical entities. Deeper symmetries correspond to more abstract eigenforms, further removed from the eigenforms of ordinary experience.

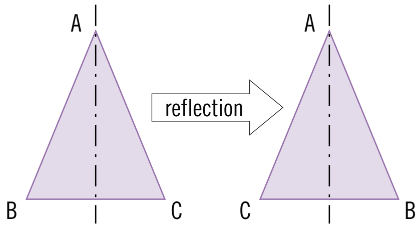

Figure 1 • An isocleles triangle is an eigenform of reflection.

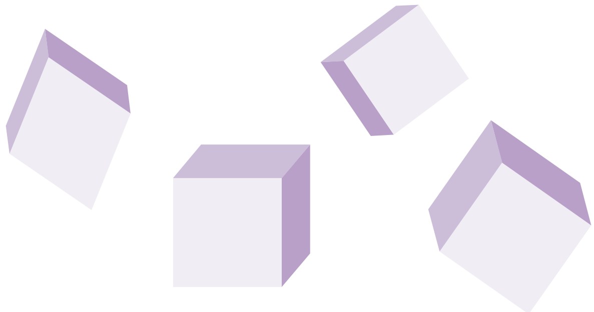

Figure 2 • The 3D structure of box is an eigenform of change in perspective.

« 8 » Albert Einstein (1956: 59) has noted that "The whole of science is nothing more than a refinement of every day thinking." We have seen that physics provides a rigorous mathematically-described illustration of how apparently objective entities correspond to eigenforms of scientific modes of observation. This is a refinement of a similar process whereby objects of ordinary experience emerge as eigenforms of our processes of observation. As Kauffman wrote, "*Ordinary objects are invariances of processes performed in the space of our experience*" (§17).

« 9 » The visual appearance of a box resting on a table, for example, may be viewed as an eigenform of visual perception. But the appearance is not an eigenform if we allow change in perspective (Figure 2). There is a deeper eigenform, however, that corresponds to the three-dimensional structure of the box. The abstracted three-dimensional properties of the box constitute a structure that is an invariant of not just a single observational mode, but a group of observational modes corresponding to different perspectives. Ordinary experience of three-dimensional objects thus emerges as a refinement of visual appearance.

« 10 » Science seeks to refine ordinary experience in several ways. For example, scientific practice requires the use of measurement devices calibrated to standardized units of measurement, and it requires that measurement procedures be capable of reproduction in different times and places. The practice of science, in effect, represents a restriction of all possible observational modes to a group of admissible observational modes. As elaborated by Joe Rosen (2008), these constraints of scientific observation may be viewed as symmetries that define science and the domain of its applicability. These observational modes of science then correspond to particular eigenforms. What is commonly viewed as the objective physical world thus appears as the eigenforms corresponding to the collection of those observational modes that are admissible by the conventions of scientific practice.

Eigenform and the first distinction

« 11 » While scientific practice is an exploration of a restricted class of refined observational modes and their associated eigenforms, we can, alternatively, seek the simplest and most general class of obser-

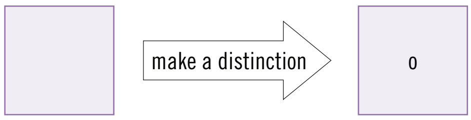

Figure 3 • Marking a page distinguishes marked from unmarked states of the page.

vation. As discussed in McFarlane (2006), many philosophers over the centuries have noted that, in order for anything to be observed or identified or experienced, it must first be distinguished. To indicate something entails distinguishing it from what it is not. Distinction thus appears at the root of all observational acts.

« 12 » Note that, in identifying distinction itself as the root of observation, we have necessarily invoked distinction. Distinction is implicit in the act of distinguishing everything, including distinction itself. The snake bites its tail. Distinction spontaneously self-arises reflexively as both an action and the result of that action. When a space is marked, the marking and the mark are identical. As Kauffman writes, "The sign indicates the very distinction that the sign makes" (§34). Reflexivity is thus at the root of every possible observation, form, or experience.

« 13 » We may ask, what is the eigenform of the first distinction? Viewing distinction as a transformation, we seek that which is invariant to that transformation. The eigenform of distinction is thus that which does not change under the act of distinction. We may view this eigenform as the context or domain *D* of the first distinction. The invariance of *D* under the action of distinction, $< >$, may be expressed as $<D> = D$. Because *D* is unchanged by distinction, it is incapable of even being distinguished from itself. It is, as it were, immune from division or duality. The act of distinction thus, paradoxically, is a non-action, creating no actual change or distinction in *D*. The form of distinction dissolves, just as mysteriously as it had apparently arisen.

« 14 » Everything is based upon distinction, yet distinction has been revealed by its eigenform to be only apparent. Yet, we may nevertheless view distinction *as if* it were actual, as a transformation that creates and exists as a stable distinction between itself and that which it is not. Viewed this way, distinction creates and indicates itself (explicitly) together with its own opposite (implicitly). Form stabilizes as duality between a distinguished state and non-distinguished state.

« 15 » Following George Spencer Brown (1969), the above may be illustrated by making a mark upon a page (Figure 3), which is viewed as distinguishing a marked state of the page (left) from an unmarked state of the page (right). The mark itself makes the distinction.

« 16 » Having constructed stabilized form, the stage is then set for the creative unfolding of endless forms within this context. Insofar as this unfolding is predicated upon the stabilization of the first distinction, we may view the first distinction as being invariant to all derivative acts of distinction and observation. It is a foundational eigenform of subsequent transformations of forms upon the page. The page itself, however, is the deeper invariant. It is the context or domain that is unchanged by even the most elemental and primitive of acts, and is indistinguishable from such acts.

« 17 » Stabilized form and structure emerge by first ignoring the apparent nature of the first distinction and regarding the first distinction as actual. This creates a derivative secondary context for further development of forms of observation and associated eigenforms. By imposing additional specific constraints upon admissible observational modes, particular worlds of eigenform arise corresponding to those constraints. Spencer-Brown's primary arithmetic in Laws of Form is one of the many possible systems of distinction that may emerge. Other examples of such an unfolding, from the first distinction to various mathematical structures, are illustrated in my unpublished manuscript "Distinction and the foundations of arithmetic" (2001), http://www.integralscience.org/lot.html. Such worlds may include mathematical worlds and experiential worlds, which themselves can be refined to evolve more complex worlds. Scientific investigation, and its associated eigenforms, may thus be seen as one possibility for the unfolding of form from the first distinction.

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Eigenform and Expertise

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> Upshot • Kauffman proposes to understand scientific thinking as including not only observations but also the act that enables their intentional use. This provides a constructivist opportunity: extending scientific thinking to gaining personal expertise.

« 1 » Louis Kauffman intends to engage "in meta-scientific activity" concerning the question of scientific correspondence (§47): whether what "does not change" in the human environment can be claimed to link or correspond to invariances in human thinking (§1, §16). The answer Kauffman proposes is full of promise. Much work still has to be done, however, to develop its consequences. This suggests an initial effort to look for areas that the proposal may help advance but does not, as yet. These areas will be referred to as "white spots," in memory of the time that Australia and Africa still contained uncharted territory.

The issue of gaining personal expertise is an example; it will be discussed below. It is at the forefront of present-day research thinking (Hoffrage & Marewski 2015; Janssens & de Zeeuw 2017).

« 2 » Mention of the Cartesian dichotomy might have highlighted the relevance of this white spot (Descartes 1998). This dichotomy entails mapping observational reports to statements such that the mappings facilitate making improved observations. For improvement to be effective one has to exclude the biases that are due to individual human motivations and preferences. This exclusion acts as a serious limitation in that humans (and other organisms) are embodied and embedded in the world – so one would expect to include at least some individual or subjective experiences. Efforts to include such experiences have led to disciplines like decision-making and (second-order) cybernetics as well as to approaches like action research (Reason & Bradbury 2001).

« 3 » Kauffman proposes another way to step out of the Cartesian dichotomy. His starting point is (second-order) cybernetics where he wishes to facilitate theory development (§32). He argues that this is possible when one uses the concept of a reflexive domain. Cybernetics is an exemplar of that concept but also its origin (§33). The concept should help overcome the limitations of the Cartesian answer (as well as those of similar answers). To this end the proposal combines two elements: (a) individual objectives and motivations that drive action, and (b) recognition of the resources to realise this action.

« 4 » Kauffman clarifies his proposal via a rich variety of examples, for example the distinction between the movements or acts of a river and what results. He compares and even seems to equate the two to the distinction between transformations in a "domain" and their "fixed points" or "eigenforms" (§2). Although this comparison has some value as an analogy, it might easily be argued to be just that: something accidental and without deeper meaning. There is no justification for the assumption that some "object for our perception" exists that corresponds to an eigenform (§17) or that "[*o*]*rdinary objects are invariances of processes* [*in*] *our experience*" (ibid). There is a gap in the way the two are equated, therefore.

« 5 » The example of the Pascal transform (§38, §40) helps to identify how to fill this gap. The distribution that results after throwing balls (or coins) through a grid may be denoted as $F(x)$ and the distribution after rolling more balls as $PF(x)$, where P is the "Pascal Recursion." It sums the frequencies of the previous distribution. The eigenform of the ensuing series of transformations is the Normal distribution. This comparison of the eigenform of the changes in the frequencies of the balls (or coins) with the Normal distribution depends on an initial act: the set-up of the experimental situation. The inclusion of this act is like that in quantum mechanics (Susskind & Friedman 2014). It stands in contrast to the Cartesian dichotomy, where this set-up is excluded.

« 6 » Without the initial act, the analogy between invariances and eigenforms would not "explain the world" (§44). With this act "the world and ourselves arise together in their creation" (§47). It allows the engendering of a "mutual creation of subject and object" (§48). These claims would have little practical meaning without the initial act. Its inclusion makes a great change. It helps to identify the need for a constructivist method to redesign what actions a human body is capable of in the here and now. In a simple form, the initial act involves selecting an objective and then searching for what experimental set-up may help to recognise and delineate the resources to realise that objective. One may think of a long-distance runner exercising to recognise and increase her bodily resources, or of someone exercising to heighten her feelings of beauty. The result, the instruction to exercise, should help to repeat the act (to serve as a memory) and to provide the condition that ensures that "new worlds will come into being" (ibid).

« 7 » If the experimental set-up allows for the appearance of an eigenform of a transformation, no personal objective remains part of that eigenform. It may thus serve as a resource to any objective so the eigenform comes near to knowledge – and helps to explain observational regularities. Kauffman summarises this result by saying that "expansion of reference [i.e., resources] leads to contraction of awareness [objectives] and contraction of reference leads to expansion of awareness" (§48). While the "expansion of reference" appears part of the scientific endeavour, individual actors may feel trapped in that endeavour if they prefer the "expansion of awareness."

« 8 » An example of this "expansion of awareness" would be that individuals prefer to act using local resources rather than knowledge (i.e., an eigenform). Local resources are those that individuals recognise when embodied in their environment and use to realise their objective(s). The activities based on such resources tend to be fast (they bypass the "expansion of reference"). If they succeed, they may also appear to be driven by intuition (Kahneman 2011). If they fail, activities that do use knowledge (and reflect an "expansion of reference") may come to the rescue. Kauffman's proposal suggests there is an alternative to such rescue. As indicated above, when one includes the act of setting up a world one may gain expertise in realising that act. It increases the ability to formulate objectives and to recognise what local resources may help to realise these.

« 9 » The "Tragedy of the Commons" may serve as an example of this second form of rescue (Hardin 1968; Ostrom 2010). Each of a finite number of actors strives to realise her personal objective by using local resources, i.e., common pool resources that while freely accessible may be destroyed through overuse (like land held in common). Destruction becomes likely when each actor only uses the common pool resources and does not recognise the way other actors provide local resources. Recognising these resources may well prevent such destruction. The eigenform that identifies this recognition appears to be similar to the Russell Operator (§36).

« 10 » This eigenform refers to the situation that observer and observed do not coincide, but act on each other in an alternating process of acting individually and collectively (§19). In this case, gaining expertise allows for an "orchestration of actions that makes an invariance" (§45). In other words, individuals may experiment to find objectives and resources and in doing so succeed to experiment collectively. This way they expand awareness and reference simultaneously (§47).

« 11 » Kauffman's proposal thus refers to the white spot of a second procedure, that of gaining expertise. Although this procedure is left implicit, it appears to motivate claims like: "all actors are affecting one another and the whole" (§31). Such mutual contributions differ from the "stable knowledge" that "is equivalent to the production of eigenforms" (§45). This difference confirms the value of local resources. They are touched upon when it is said that "[t]he world itself is affected by the actions of its participants at all levels" (§45).

« 12 » Kauffman refers to cybernetics as "defined in terms of itself." It focuses on the observer observing herself as an eigenform of the negation operator. This suggests that he recognises the white spot: the focus on observation implies that the focus on action is neglected so further work is necessary. The same type of neglect is exemplified by the focus on "theory" in the area of evolution (Stadler 2016). Kauffman's proposal suggests that in both areas one should emphasise the process of gaining expertise – as part of a "struggle." Organisms have experimented for a long time to (collectively) gain expertise in recognising and using local resources to deal effectively with their environment. Knowledge as a resource is a late arrival.

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Author's Response

Eigenform, Action, the Continuous and the Discrete

Louis H. Kauffman

> Upshot • I discuss the epistemology of action and its relation to the continuous and the discrete, relative to eigenform, eigenbehavior and the deeper structure of eigenforms in relation to the construction of our personal and apparently objective worlds.

« 1 » I will begin with a succinct summary of my point of view about eigenform and then explain how this point of view is related to the many questions and statements given by the authors of the commentaries. In his §6, **Gerard de Zeeuw** remarks that the initial act of transformation is very important for the understanding of eigenform. I agree and will discuss the point now in general, and later quite specifically. An eigenform is a fixed point of a transformation. A transformation is an action that occurs in some domain. Thus, an eigenform begins with an action, and an eigenform is invariant under that action. The notions of both action and invariance occur for some observer who may be part of or identical with the domain that is observed. What is invariant, the eigenform, is not necessarily part of the original domain. The eigenform may be a new entity that has arisen for the observer in the course of interaction with the action or transformation that originated the process leading to the eigenform.

« 2 » Without action, transformation, there is no possibility of an eigenform. It is in the presence of change that the observer can understand, indicate or find constancy. Without some form of constancy, there is no way to observe action. Action is seen against some reference that is regarded as constant.

« 3 » We have indicated the epistemology of eigenform by using a symbol T for a transformation. In this symbolism, an entity X is transformed into a new entity $T(X)$. This gives rise to the possibility of a recursive application of T upon an initial state X and the possibility of an excursion into mathematical formalism. The simplest version of this formalism is the writing of the composite symbol $E = T(T(T(\ldots)))$ that can be taken to mean "an indefinite application of T upon an unknown state" or "the action of T upon itself." In the latter interpretation I can interpret that action of T upon itself as being unchanged by the action of T, just as my thoughts of myself do not change the essential notion of myself (and in fact may support it). In this form we have it that E is a fixed point for the action of T, and so E can be seen as an eigenform for T. In this way any action can give rise (by acting upon itself) to an eigenform. Many cognitive eigenforms have exactly this pattern and it is the case that many objects in the worlds of our perception can, in the last analysis, be identified with actions that are associated with them.

« 4 » There is always the possibility of examining a recursion that is associated with an action. For example, the North Atlantic Ocean acts upon the coastline of Britain over many years so that the coastline becomes an approximate (it is still changing) eigenform for this action. Once approximation appears we enter into mathematical considerations where matters of limits can be quite subtle and the many conceptual domains of mathematical thought need to be examined once again. One of the largest themes related to eigenform then becomes the relations of the continuous and the discrete. These themes are touched upon in my essay with the description of the binomial distribution and its limiting form, the continuous normal curve.

« 5 » **Jean Paul van Bendegem** takes me to task for dealing too loosely with the limiting properties of the binomial distribution. I plead guilty to this accusation! I did not intend to write a mathematical article. My intent was to point out that is well-known to those who study probability and statistics: that the *form* of the normal distribution is approached by the *form* of the binomial distribution. I was not asserting any facts about the point-wise limit of the one and the other. The forms of these distributions can be seen from their graphs and from experiments with balls dropping through lattices. But to actually analyze the limits of the binomial distribution in relation to the normal distribution is a technical mathematical task that was not my intent in the target article. I am tempted to include here some of the details that I like about the mathematics of these limits but I shall forgo the pleasure. The point I want to make is that the two distributions, one continuous and the other discrete, are related not just by form but also quantitatively in that for a toss of a large number of coins we can make quite accurate predictions about their behavior by using the continuous normal distribution. How this can be done is the subject of basic statistics and is a hard-won mathematical subject.

« 6 » **Van Bendegem's** objection points out that if we are to make full use of the concept of eigenforms we shall have to respect the mathematical structures that lie behind them. This does not only apply to the binomial distribution but even to the most general eigenform that I have indicated in §3 above. To make mathematical sense of $E = T(T(T(\ldots)))$, we need to work in a number of directions, as I have in previous papers, using notions of infinity or using notions from the lambda calculus of Church and Curry (Barendregt 1984). **Van Bendegem** objects to the idea that there might be domains where there are always eigenforms. One could agree in concrete mathematical contexts. For example, the Brouwer fixed point theorem: A continuous mapping of a closed disk to itself has a fixed point. If we did not take the closure hypothesis for the disk, this theorem would be false, and it would be inaccurate and uninformative to claim otherwise. But in a reflexive domain D where D is in 1-1 correspondence with the actions of the elements of D, then every element of D has a fixed point by the beautiful construction of Church and Curry: Let A be an element of D. Define $Gx = A(xx)$. Then $GG = A(GG)$. This beautiful argument works under conditions of reflexivity that are not available when we consider the Brouwer fixed point theorem. The moral of the reflexive domain is that it describes a certain kind of cybernetic environment where every element is also an actor. It is an idealization of the place where action and object are identical. This is an important conceptual domain for cybernetics and needs to be understood more than ever in the present social and political world.

« 7 » **Van Bendegem** also remarks on the amazing behavior of the differential equation $df/dt = f(N - f)$ in relation to its discretization. The discretization behaves chaotically in a complex manner that the differential equation does not display. Important behavior of the discrete system is lost in taking the usual limit. What is the eigenform here? This question requires study. Many people have regarded the chaotic part (which has the *form* of the famous logistic recursion) as something to be studied in its own right, something that is lost when using the differential equation. In that case the full complexity of the logistic recursion is the eigenform, and one can devote one's life to it. No one said that eigenforms were simple. It is what is produced by a certain action. *An eigenform can be what is produced by a certain action*. The invariance is the fact that it is produced by a certain action. A given action produces a determinate result. $Y = F(X)$. You start with X and get Y. Define $T(A) = F(X)$ for any A. Then if $A = Y$ we have $T(Y) = F(X) = Y$. I apologize for not making this fundamental eigenform clear in my essay. Any repeatable producible result is an eigenform. All of the results of science are eigenforms. It is a tautology and it is a *significant tautology*.

« 8 » **Van Bendegem** ends his wonderful criticisms with the injunction to examine the relations between the countable and the uncountable in terms of eigenform. This is a favorite of mine and I cannot resist. Let Ax mean that "x is a member of the collection A." Suppose that we have a method to associate to every member of a collection A a subcollection of A. That is, suppose that for a in A we have F(a) a subset of A. Now define $Cx = {\sim}F(x)x$. This means that x is a member of C exactly when it is not the case that x is a member of $F(x)$. Could it be that C, being a subset of A, could be of the form $F(z)$ for some z? Let us try. Then $F(z)x = {\sim}F(x)x$ for any x. So, we let x be z and find that $F(z)z = {\sim}F(z)z$. We have produced a fixed point for negation ~! But in the world of standard logic and sets it is not allowed that negation should have a fixed point. Therefore, C is not of the form $F(z)$ for any z. This means that the number of subsets of A is bigger than the number of elements of A. If A is countable, then the number of subsets of A is uncountable. Uncountability has arisen from the *denial* of an eigenform! In standard mathematics we have reified the uncountable infinities and then found that they lead to fantastic properties and conundrums. It is a widely accepted mathematical viewpoint (the normal "reality" for nearly all trained mathematician) that arises because we do not allow a certain fixed point. In standard mathematics it is a given that there is no fixed point to the negation operator. I will stop with this remark. I am not saying that we should devalue uncountable infinities because they arise from a denial of a certain form. I am saying that eigenform is always there whether you take it or do not take it. For the mathematicians, there is a belief that the systems produced under these conditions are consistent. If it were to be shown that denying a fixed point for negation would lead to a contradiction, then there would be a shift of very large proportions. I mention these matters to show how "realities" arise for groups of scientific workers. We could make similar examinations in other natural sciences. In this sense of the use of the word reality I do not take it to be forbidden. Each reality is an eigenform with deep internal reflexive structures involving groups of persons acting out the limitations of the form.

« 9 » I thank **Karl Müller** for the many encouraging remarks and would like to say that I am myself very heartened by his concept of endo-science as expressed in his §7. In endo-science we can take a given scientific field and include within it not only the "science" as we come to know it, but also the scientists and their larger relationship in human society. I think that those of us who wanted to become scientists when we were young were keenly interested in endo-science. We devoured biography about the scientists and we wanted to know what it was like to be such a person. There is a popular interest in this dimension. Witness books such as "The Double Helix" by James Watson (1968). And if we keep observing science and the scientists then we shall see how all science is a human endeavor and yet can produce apparently objective and new knowledge about our worlds, perhaps always at the expense of insisting on certain (eigen) forms and denying others. Of course, there is much more in this domain: We want to know. We want to produce new results. We want to find out about their consequences. We want to have responsibility for the consequences. We cannot have everything, but we must not remain in our ivory tower. It is neither safe nor intelligent, nor ultimately fun to do that any more. In my opinion, the most satisfaction comes in reaching for the largest possible observation.

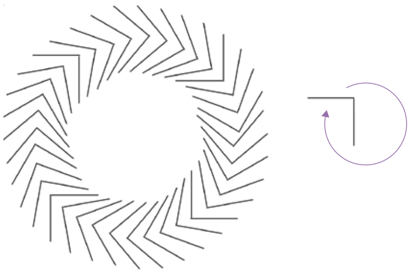

Figure 1 • Circular reentry with circular symmetry.

« 10 » In his §11, **Tom McFarlane** brings the theme back to the fundamental matter of distinction. I agree entirely. A favorite point that I like to make is that the mark of George Spencer Brown in his seminal book *Laws of Form* (Spencer Brown 1969) is self-referential since it is seen to make a distinction and it stands for the (act of) making a distinction. Thus, the mark is an eigenform of itself. In the formalism, we do not have $<< >> = < >$. It is not an eigenform in *that* sense! One needs to understand that the mark $< >$ designates itself without writing extra marks to indicate that self-reference. Doing that act of understanding one arrives at a fact that cannot be expressed without complication, because the very act of expressing it obscures its nature. For example, we could write "$< > P < >$" where "P" stands for the act of pointing. But then in this expression there are two marks, one on each side of the P and the reader must understand that each stands for the same mark.

« 11 » When we write one mark $< >$ and remark that this mark stands for itself, we have accomplished a thought that becomes more complex when it is articulated than it is when it is thought. This is my opinion. The point of this discussion is important for eigenform, for ultimately eigenform occurs in the boundary between understanding and articulation.

« 12 » **McFarlane** points out the beauty and utility of the concept and action of symmetry in eigenforms. This is a deep and wonderful remark. Let us begin to understand symmetry. In the simplest evocation of eigenform, an infinite composition such as $E = <<<<<\ldots>>>>>$ so that $<E> = E$ we have a precise symmetry in the infinite form E that shifts it once inward. This is the same pattern as the symmetry of the integers defined by $F(n) = n + 1$, but note that for the integers F is defined for both positive and negative numbers and the mapping $F: Z \rightarrow Z$ is both 1-1 and onto. Here I take Z to be the set of all integers $Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. A corresponding E might look like $EE = \ldots<<<\ldots>>>\ldots$ so that it never stops, inside or outside. This EE does not have a "0-place" but it is symmetrical both inside and outside. Another example of symmetrical eigenform is illustrated in Figure 1 where we have a circularly reentering mark.

« 13 » Once symmetry enters, as it does in the circular eigenform of Figure 1, we find that a number of themes come forth. One theme is that mathematical language is not as expressive as the geometry of the form itself. In this way, the geometry comes forth as a new language in which to express symmetry and relationship. Another example of this tendency is exemplified by the Fibonacci Form $F = <<F>F>$, one of the simplest eigenforms other than the simple reentering mark $J = <J>$. As we know, there is much development of geometry related to the Fibonacci form and its appearance in nature, from the Nautilus sea shell to the sunflower. In all cases the form is clothed in the beautiful symmetry of its enlarged context. We see that symmetry and significant context are closely related, nay inextricably intertwined with each other. **McFarlane** turns to this theme in §8. What he writes points precisely to the role of context and how "deeper" eigenforms will arise that correspond to invariances within a given context. The theme continues into the way physics holds elementary particles in terms of their symmetries and the way as in §7, geometers and topologists continue to look for the deepest and simplest descriptions of the essence of the "objects" of their study. Deeper eigenforms are often the most difficult to express, and can give the most scientific satisfaction when even a partial answer is found.

« 14 » In §4 of his commentary **De Zeeuw** says that "There is no justification for the assumption that some 'object for our perception' exists that corresponds to an eigenform." And he suggests that this is a gap to be filled in the epistemology of eigenform. I do not exactly disagree with this statement. My reaction is this: If there is no justification that some object of our perception exists then it is *just* an eigenform and nothing more. Hallucinations and certain optical illusions fall into this category. Some person might attempt to convince us that our perceptions are populated only with eigenforms and there is no justification for any other kind of existence. But what about the lamp on my desk? Can I justify its existence? I can investigate how it is and how it works and of what it is made. This is a nearly endless investigation. It includes the chemical and physical composition of the matter of the lamp and its electrical properties. It includes the history of its invention, and this leads out into the history of all technology. We can continue, and eventually I find that the story of what that lamp is, is the story of what the whole universe is in which I live and breathe, including all my actions, reactions, thoughts, hopes and fears. This is not a metaphor. It is literally what becomes of the investigation into what any given thing is in one's world. All that becomes the eigenform of the lamp, indistinguishable from the lamp itself and indistinguishable (except at the cost of denying certain eigenforms) from the observer who in this temporal context is taken to be myself. The quest for the identity of any "ordinary" object such as the lamp on my desk leads outward into an investigation of the entire world of the observer. After all that, the object is not separate from the observer or from that world, but something of great use has been understood. The actions that generate this investigation take us out of the skeletal abstraction and into the whole world of possible and actual relationships. (If the reader is still incredulous that cosmology would enter into the understanding of an ordinary object, consider the fact that in a rotating bucket of water, the liquid will move up along the sides of the bucket. Consider that reverse point of view with a stationary bucket and the whole universe rotating about the bucket. The result must be the same by the relativity of the description. In this way Ernst Mach (Barbour & Pfister 1995) concluded that the effect of the water moving up the sides is due to the gravitational effect of the entire universe in relation to the motion of the bucket. In a universe empty except for the bucket, there could be no such effect.

« 15 » In his §6, De Zeeuw remarks that the initial act of transformation and the initial state that is transformed is all-important for the actuality of an eigenform. I agree fully, and refer to my §§1–3 in this response for my full response on this point. I have placed these paragraphs at the beginning of this response because they apply to everything that is said here in a multitude of ways. We can view the inception of an eigenform as a dynamical invocation of the inception of a distinction. *At first what is acted upon and what is the actor are not differentiated from each other.* Then as time goes on (time itself emerges from the act of distinction) the roles of actor and actant (the one acted upon) begin to be separable. At last there comes a time when actor and actant can be at least partially separated, and one can say "This acts upon that." "I speak and they listen." "The electron hits the screen." Once it is understood that the beginning of such processes is undifferentiated, it becomes easier to understand how certain situations arrive at their distinctions in different orders from what we have been brought up to believe is the norm. For example, in a double-slit experiment we have been taught by Niels Bohr and Werner Heisenberg to say "The electron hits the screen" *only after the observation of an event on the screen has occurred.* In our grammar from childhood, to say that an electron hits the screen is to give that electron existence prior to the time of the meeting with the screen. This is forbidden speech for Bohr and Heisenberg, who would construct an epistemology of eigenforms for quantum mechanics where the invariance only begins at the point of actualization. It is a bold attempt to change the language of action, and in it we see once again how the descriptive worlds of science are built on the construction and observation of eigenforms, and equally on the forbidding or avoiding of other possible eigenforms. Once these contexts are understood, we can speak about action. Before such access to context, the notion of action is fraught with our assumptions derived from the status quo.

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