person to think that they have misunderstood the other (Glasersfeld 1995).

« 12 » This issue is relevant to the Borg et al. framework in that it allows for the distinction between when a student makes an accommodation (i.e., act of learning) in relation to her understanding of the teacher's (or other students') understandings versus when a student makes an accommodation in relation to her own mathematical understandings. Both cases involve accommodation and may take place in concert with each other, but they can also occur independently.

« 13 » Throughout the data analysis I wondered whether and when students were making accommodations with respect to their understanding of the teacher's (or other students') understandings versus making accommodations in their own mathematical understandings. For example, in Protocol 3 I wondered whether the students had fundamentally changed their mathematical understandings in some way, or whether they had changed their understanding of the intended meaning of the teacher's question. This issue is important in characterizing the kind of learning that may be occurring in classroom interactions. In particular, it is instructive for teachers (and researchers) to consider how interactions among students and teacher could attain the status of inter-subjective agreement without students fundamentally changing their mathematical understandings. Finding evidence for this, and contrasting it with when students make accommodations in their own mathematical understandings is an important nuance that Borg et al. could address more explicitly in future work. **Do Borg et al. see this distinction in the data excerpts that they share? If so, how? (Q4)**

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Facilitating Constructivist Principles in Using Apps: Moving from Class Video to Community

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> Upshot • The target article offers a method for teachers to reflect on their constructivist approach in classrooms. This commentary suggests ways to augment the approach for use with groups of teachers.

« 1 » The authors of the target article used video recordings to analyse mathematics teaching with a view to facilitating analysis of one author (Philip Borg) using constructivist principles in class using a maths software programme (Grid Algebra). They described the tensions created by the need to teach a set curriculum to pupils who vary significantly in terms of their readiness (§9). The introduction refers to the debate about use of the phrase "constructivist teaching" since constructivism has at its centre the idea of learning. A first point for the authors is to ask them to extend their brief mention of the debate concerning constructivist teaching (CT). For example, the issue of the role of the teacher in constructivist learning is admirably illustrated in the target article (§1). **However, what are key differences they refer to between CT and progressive methods? And again, would the authors agree that while CT is not one particular method, it might be the case that individual teachers can be expected to develop their own styles of CT? (Q1)**

« 2 » The article shows in detail how the use of video facilitated the teaching author's awareness of his own strengths and weaknesses in his own classroom work. In what follows I want to discuss some implications I find in this work for helping teachers to engage reflectively with their own practices with their pupils.

« 3 » The authors' perspective in using Grid Algebra (GA) software begins with the framework of relations between mathematics, negotiation and learner (M-N-L) (§4). This raises the possibility of discussing the approach taken from the "teacher perspective" and in addition moving beyond this perspective to consider the social interactive level of the mathematics community in the classroom. The focus of the target article is principally on the M-N-L framework (§2) emphasising teacher-learner interactions rather than including learner-to-learner comments and interactions. **Would it not be useful to further analyse these videos to examine ways the teacher and pupils facilitated or participated in co-operative learning? (Q2)**

« 4 » One of the contributions of the target article is that it allows teachers and teacher educators to appreciate the complexity involved in balancing set curricula with fixed goals with widely varying pupil/student understandings of the material in an educational setting. This tension is clearly present in comments made concerning the teacher's difficulties maintaining a constructivist stance in class. In particular in (§§38f) where the teacher regrets not being able to pick up on pupil initiatives, it seems to me that there may be ways to acknowledge pupils' different perceptions without allowing these perceptions to derail the focus of the class conversation. On the basis of my experience observing and supervising student teachers in classrooms, I am aware how easy it is to miss opportunities to capture moments when individuals might have been helped, had the teacher noticed or acted. Yet, at the same time, classes need a flexible focus where the teacher can notice an opportunity and decide whether to stop or return to it later. **Perhaps the important thing is that the pupil knows the teacher noticed and that later the teacher follows up with the pupil? (Q3)**

« 5 » There will always be moments when teachers have to decide whether to follow the lesson plan or to stop and help individual students. Constructivist theory prioritises the moments when pupils show that uncertainty may be the opening to learning something new. Perhaps this is part of the art of teaching, knowing when to stop and digress and generate a context where lasting learning will occur.

« 6 » One question I had on reading (§§11f) concerns the issue of moving from understanding how to use the software to

learn that multiplication is a form of successive addition, and conversely to learn division is a form of successive subtraction. I have a memory of a maths class when I was making this link with some difficulty! I knew how to divide and subtract, but there was another awareness that I was having difficulty constructing, namely the awareness that these different operations were connected. Today I might call this a higher-order connection. Aged about 10, I was missing this awareness that dividing 2 into 5 and obtaining the answer 2 with 1 remaining, was precisely the same as taking 2 from 5 successively (twice) and having 1 left over. So turning this memory into a question for the author, how do you avoid ensuring that pupils using GA software with row and column manipulations to find answers will also notice these deeper mathematical relationships between addition, multiplication, subtraction and division? Or putting it another way, is there a danger that the pupils will use the software like a calculator and not develop a feel for whether answers are correct or not? Is there a danger that pupils will not move from learning about the tool at a simple level to learning with the tool at different levels? So, would it be an advantage for teachers to design tasks for pupils to work in groups using GA to ensure these relationships are well understood and therefore ensuring the advantages of learning through social cooperation (Gash 2014: §27).

« 7 » If a group of teachers could arrange to view videos of one another's classes together and share their views this might be a productive way of promoting professional development. Recently in *Constructivist Foundations*, Karen Brennan (2015) discussed the evolution of teachers' skills using technology. While Brennan focused on constructionism and so on the values of learners designing and making things in the classroom, it seems to me that the social processes she described would be one excellent next step in the author's exploitation of the experiences described in the target article. Another way of using video to promote professional development is described by Jana Visnovska and Paul Cobb (2013). **Perhaps the authors are already engaged in these types of activities? (Q4)**

« 8 » The moments of uncertainty or frustration the author describes in the target article (§22) are moments that have been commented on in Therese Dooley's work (2010). Dooley also focussed on learner epistemic actions including recognising and constructing; these actions resonate with bridging between the learner and the features of negotiation (§18). These commonalities in the approaches of mathematics teachers coming to terms with the tensions between emphasising learning as constructivists and being responsible for set curricula demonstrate the importance of methods such as these that facilitate self-reflection. A strength of the approach in the target article is that the M-N-L tool offers a way of examining dynamic changes in both teacher approach and learner construction.

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What Do We Lose If We Abandon Constructivism?

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> Upshot • While I appreciate sensitive teaching approaches to students' learning mathematics using Grid Algebra software, I am unconvinced that the approaches described are constructivist in nature. To make further progress along the lines described by the authors a clearer articulation of its constructivist foundations is needed.

« 1 » In their target article, Philip Borg, Dave Hewitt and Ian Jones discuss the teaching and learning of mathematics using a software tool Grid Algebra. The teacher is also a researcher, collecting and analyzing data from his interactions with students. In the article, episodes of dialogue illuminate and provide insights into the teaching approach, the ways in which the teacher thinks about mathematics and the learning of his students, and his own approaches to promoting/supporting their learning. A fundamental tenet of the teacher's practice seems to be his *sensitivity to students*, the forms that this takes and the tensions that arise. The authors write that the teacher is working from a *constructivist* perspective (*radical* constructivism is mentioned) and they talk about the teacher's "sensitivity" to his constructivist principles. Therefore my commentary starts with the meanings of *constructivism*, *constructivist teacher* and *constructivist teaching*, which are exercised throughout the article, and how these relate to the goals, actions and reflections of the teacher as expressed through the episodes.

What constructivism is – and the meaning of constructivist teachers and constructivist teaching

« 2 » At the beginning of the article, the authors write that "constructivism is a theory of learning, not of teaching" (§1). I certainly agree with this. However they refer, without further elucidation, to Martin Simon (1994) and Arne Engström (2014), whereas I might have in mind alternative sources such as Paul Ernest (1991) or Ernst von Glasersfeld (1983). Would these four sources agree on the theoretical groundings for stating that "constructivism is a theory of learning not teaching" (§1)? I make the point here that the nature of the theory of constructivism, as a basis for this article, needs more than a passing reference (or two). Here, the reader is left to infer the authors' theoretical groundings. For example, readers are given some hints: a constructivist epistemology contrasts with a *realist* epistemology and is compatible with *progressive* modes of teaching in which "a second-order observer might consider [exposition] 'traditional' and 'non-constructivist'" (§5B). Unfortunately, the terms *realist* and *progressive* are not discussed either.

« 3 » In Jaworski (1994: 15–25) I explained in some detail my own understandings of constructivism and its relations to the learning and teaching of mathematics. On the one hand, if the constructivist epistemology of the teacher accords with *radi-*