Reasoning in a Dynamic Animation Environment

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> **Upshot** • Parnorkou and Maloney describe how a dynamic animation environment, Graphs 'n Glyphs, supported fourth-grade students' understandings of translations and rotations. Two elements were critical in their teaching experiment: the design of the software and tasks. This commentary focuses on the decisions that they made and possible implications they had for students' reasoning.

« 1 » The importance of geometric transformations in developing students' understandings of geometry and function has been acknowledged for some time and this topic has been included in the K-12 mathematics curriculum in the United States for several decades (Coxford 1991; National Council of Teachers of Mathematics1989, 2000; Sinclair 2008). In addition, the study of geometric transformations is amenable to the use of non-technological and technological tools (Coxford 1991; National Council of Teachers of Mathematics 2000). Nicole Panorkou and Alan Maloney share interesting insights from their research about the ways in which a dynamic animation environment, Graphs 'n Glyphs (GnG), supported fourth-grade students' understandings of translations and rotations. Two elements were critical in their teaching experiment: the design of the software program and the design of the tasks. My commentary will highlight important decisions the researchers made in the design of the software and tasks and discuss possible implications these had for students' reasoning.

Software design

« 2 » When designing software for geometric transformations, one must carefully consider the ways in which teachers and students interact with the tool, the information the tool provides, modifications teachers and students can make, and a history of actions and how they are displayed (Under-

wood et al. 2005). In this regard, GnG does a great job, displaying a history with the timeline. The coordinate grid background is another critical and important feature. With the coordinate grid, students build on their experiences with number lines for adding and subtracting to consider horizontal, vertical, and, finally, diagonal translations. It was clear that students were able to use the grid to perform translations and rotations. However, it was not clear how this interface scaffolds students as they transition from thinking about translations performed in a vertical and horizontal direction to thinking about translations defined by a vector.

« 3 » To examine how other software programs represent vectors, Cabri Geometry II Plus, Geogebra, and The Geometer's Sketchpad were reviewed. Cabri Geometry II Plus requires the selection of the object and then the selection of the vector that defines the translation.

« 4 » Geogebra requires users to interact with the software in a similar way as Cabri. The Geometer's Sketchpad allows one to translate by a polar, rectangular, or marked vector (Figure 1).

« 5 » What is interesting with this interface is that students are able to enter a negative value for the horizontal or vertical fixed distance to move left or down. Although the GnG interface is most closely related to The Geometer's Sketchpad, rather than using distance in centimeters to specify the horizontal and vertical components of the vector, units determined by the coordinate plane are used. This coordination is nice and builds on students' understandings. However, despite the claims of Parnorkou and Maloney, it is not clear what students will come to understand about vectors as having magnitude and direction, or what they will understand about distance when negative values are entered. For example, in excerpt 1 of students' work with the tool, the researcher asked students to move a square eight spaces to the left. The student stated that, "minus means go down" but he entered –8 to move left, not down. He seemed to understand "go down" as "back or take away." The use of the negative sign in this manner could be problematic and may not support students' understanding of the magnitude or direction of a translation vector. Negative values are not used to describe magni-
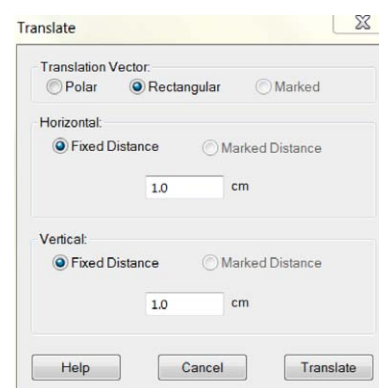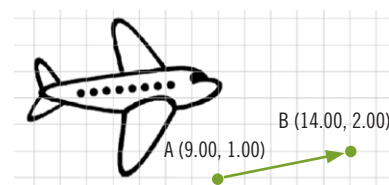

Figure 1 • Dialog box for translations.

Figure 2 • Use of a vector to indicate a translation.

tude nor are they used to describe direction. Perhaps a task could be designed that places vectors on a coordinate grid to develop students' understandings of translation vectors. Several comments about the design of tasks are provided.

Representing vectors

« 6 » To build on students' understanding of translations as moving in a vertical and horizontal direction, a vector described with coordinates for its head and tail might be introduced (Figure 2).

« 7 » This representation is related to the number line and could move students to thinking about direction up, down, left, and right without using the negative sign to refer to the left and right direction. Another option would be for the dialog box to have four quadrants to allow students to move down and left.

Selection of objects

« 8 » It is also important to note that the objects students are provided to transform are important to consider. Students may

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not notice anything has changed if they are provided with a square to rotate 180 degrees about its center or reflect about a vertical or horizontal mirror line. This applies to other objects that have symmetry. The dog and fish provided in the GnG environment were likely more helpful to students than the star. It is unclear why these images were not selected to be more "real-life" given the animation environment context.

Referent points

« 9 » It is also imperative that students become familiar with the language of preimage and image points and that students are provided with corresponding points. It was clear that students learned that the "points had to match." Excerpt 3 shows how students were able to match the two stars, but had difficulty matching the two houses. It is important to note that point A was marked on the preimage and point B on the image was not marked. Students might have assumed point B was located where it was labeled. It is also interesting to note that the preimage (the house on the right) included its own axis but this was not included on the house on the left. These differences may have contributed to students' difficulty. It is unclear whether points A and B were intended to be corresponding preimage and image points or perhaps they were intended to be labels on the objects. This is a critical distinction that would need to be discussed with students. Also noteworthy is that the sequence of actions students performed on the house task might be reflective of a reactive strategy, rather than a proactive strategy they seemed to use with the star task (Hollebrands 2007). Perhaps the difference was related to the direction of the translation, the object provided, or the interactions the students had with the researcher.

Summary

« 10 » Overall, the research report discusses interesting ways students reasoned about translations and rotations. It is important to consider what students notice and observe and how these may be related to the design of the software and tasks, and their interactions with each other and the researcher.

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Body Syntonicity in Multi-Point Rotation?

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> **Upshot** • Parnorkou and Maloney's article presents an interesting, well-structured and clearly described study of children's reasoning about mental rotations. Specifically, Parnorkou and Maloney deploy the microworld Graphs 'n Glyphs, and use it as a "window on thinking-in-change" as they observe and interview children who use it. Reading the article raised a few questions for me about the role of body syntonicity in learning about rotation of geometric shapes, and I wonder where (or if) the authors feel these foundational concepts fit in with their research.

« 1 » The target article presents a study on the development of children's geometric transformations while using Graphs 'n Glyphs (GnG). Previous research has primarily focused on quantitative assessment of children's scores on GT activity tests, and has been done with older learners. The authors argue that the quantitative approach fails to capture not only qualitative elements of this thinking, but also the qualitative properties of the *change* in this thinking. The study borrows from Richard Noss and Celia Hoyles and uses activities designed in microworld as a "window on thinking-in-change"-approach to understanding how children develop their thinking about geometric transformations. The study then recounts how prior research helped inform them of the design of activities, focusing on translation and rotation (and scale and stretch, not in this article), that would be able to engage younger learners at an appropriate level. Finally, they specify their research questions, focusing in part on *how* young learners think with GnG, and in part how GnG functions as a research tool for studying this particular kind of thinking.

« 2 » The rest of the article describes first their study, and then a set of excerpts from the researchers' interactions with two students, and finally addresses their two focused research questions. First, let me start by saying that I think that it is an interesting study and article and that it addresses some central research questions in constructionism. First, how do we design microworlds that simultaneously act as inspiring and fun learning environments *and* as elicitors of externalized thinking? Straddling the combined agenda of furthering both children's understanding and our understanding of their understanding is an ever-present challenge in constructionist research and design. The authors do this well by having students solve tasks that encourage explicit dialogue about the hows and whys of solving each of the problems. Second, since we view knowledge as constructed from prior knowledge through reflection on the construction (and manipulation) of shared objects, how do we design microworlds that provide learners with these objects-to-think-with and with relevant primitives for performing mental, physical, or computational operations on them? The combination of the design of the particular activities and GnG seemed to provide students with meaningful ways of discussing and performing the manipulation of shapes.

« 3 » However, two things about the design of primitives that stood out to me were the decisions to, first, have a default direction in rotation (counter-clockwise) and to use minus as "the other way," and, second, to use + and – in translation as meaning respectively up and down or right or left. This is somewhat of a departure from Logo (and later Logo variants) and their body-syntonic approach to encouraging learners to think about objects as having a "heading" and being able to turn left or right and move forward and backward. Clearly the present design was meaningful to students, and it even

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