cedure-mastering learner whose behaviour has only been trained is more hidebound, more routine in the actions they enact.

« 8 » Fostering and sustaining a constructive stance to learning involves more than providing engaging tasks, more than encounters with pervasive mathematical themes, more than experience of one's own use of natural powers in a mathematical context. It requires immersion in and prompts use of a vocabulary that captures those experiences and enables learners to become aware of what has been effective and what has not, not only at the end of a piece of work, but throughout. It is the construction of a personal narrative, with on-going improvements and refinements, that constitutes learning in the fullest sense of the word.

John Mason is a professor emeritus at the Open University and a senior research fellow at the University of Oxford, Department of Education. His interests span problem solving; mathematical thinking; calling upon and developing learners' powers, particularly generalizing; the use and role of mental imagery in teaching and learning mathematics; the nature and role of attention in teaching and learning mathematics; the discipline of noticing applied to research as well as teaching and learning. Homepage: http://www.pmtheta.com

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Authors' Response:
Let's Cross that Bridge… but Don't Forget to Look Back at Our Old Neighborhood

Eirini Geraniou
& Manolis Mavrikis

> Upshot • This response addresses the main points from the three commentaries, focusing particularly on additional terms and concepts introduced to the bridging metaphor. We further clarify our call for future research in the area and conclude with reflections about the practical implications emerging from our target article and the commentaries.

« 1 » In her commentary, **Nicole Panorkou** explicitly reminds us, among other relevant literature, of the concept of *contextual neighbourhood* from Pratt & Noss (2010), which of course permeates our research due to the direct and indirect influence of the research of the authors in our work. In §3, **Panorkou** challenges us to define our context explicitly. Revisiting Pratt & Noss (2010), we are reminded that in design-based research, the determination of contextual neighbourhoods is sometimes implicit, both in the case of software design, as in eXpresser, and in the case of task design, as in our paper-and-pencil activities. The challenge with making context explicit is that it is in the eye of the beholder. We therefore have at least three contexts to elaborate on — researcher, student, and teacher or schooling contexts. In brief, the framework of algebraic ways of thinking (inspired by Seymour Papert's 1972 reference to mathematical ways of thinking) gave us, as researchers, the lens through which to examine students' activities and learning as well as a way to map those to the teacher and the schooling parlour (e.g., in our case, to the national curriculum). These contexts of course overlap, and perhaps the distinction is mainly academic, but we are primarily interested in the context as perceived by the students and its influence in the knowledge or ways of thinking that they develop.

« 2 » We see therefore the rest of **Panorkou**'s review as a call for future research, particularly her excellent suggestions, in §§5f, on how students' learning trajectories between contexts, facilitated by bridges, can be studied. We see our article as a first step towards this investigation. The "bridging" activities were a design-based research outcome after carrying out a number of studies; they therefore served a purpose in the research context rather than the object of investigation itself. We agree, however, that future research should be structured in ways that bring out the dynamic nature of the bridging activities and (sticking with the metaphor) help investigate what situated abstractions (Hoyles & Noss 1992) or other learning takes place on the two sides of the bridge.

« 3 » Along the same line of thought, **Ian Jones**'s review first brings to our attention a recent paper by Dave Hewitt (2014) that can also help in future research by thinking in terms of scaffolding and fading. In §2, he raises an important question that has troubled us and the team behind the original MiGen project that designed the eXpresser micoworld and its tasks: Is the time investment in scaffolding students through one microworld, designed with specific algebraic ways of thinking in mind, worth the trouble?

« 4 » We think that an answer to this conundrum comes on the back of more than 40 years of research in constructionism and endless debates since. Avoiding opening a can of worms in such a short response, our other papers on eXpresser have demonstrated its potential (e.g., Mavrikis et al. 2013), and **Jones**'s eloquent summary of functional and structural approaches in §§5f provides claims towards the potential of a microworld to support flexibility. Additionally, we rely on anecdotal teacher reports and our experience of the potential of using eXpresser and other microworlds in so called "blended-learning" scenarios, recently popularised by advocates of "flipped learning." We have seen first hand the potential of giving students eXpresser homework or group projects that can act as substrate for a teacher-led plenary, or subsequent engagement with traditional algebra in the classroom.

« 5 » An additional answer to the point above lies between the lines of the third commentary by **John Mason**, whose research on mathematics education and his contributions, in particular on algebra learning (Mason 2005a; Mason et al. 1985), have heavily

influenced the design of eXpresser and its associated tasks. Mason refers to the engaging potential of digital technology that can paradoxically lead to a situation that is not conducive to learning *per se*. Mason invokes George Polya's "looking back," which so elegantly frames the aim of our bridging activities. We want to help students to take a step back from the microworld in which they have immersed themselves and remember to learn.

« 6 » Putting all the commentaries together brings us to the title of this article. Engineering (in the sense of Cobb et al. 2003) eXpresser activities interspersed with bridging activities at appropriate time points can answer Jones's question with respect to efficiency, achieve Mason's call to encourage students to capture those experiences and become aware of their work by looking back to their interactions throughout the eXpresser tasks, and achieve what Panorkou saw as expansion of contextual neighbourhoods.

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